Analysis Model of a Small Scale Counter-Rotating Dual Rotor Wind Turbine with Double Rotational Generator Armature

Ercan Erturk*†, Selim Sivrioglu**, Fevzi Cakmak Bolat**

*Bahcesehir University, Mechatronics Engineering Department, Istanbul Turkey
**Gebze Technical University, Mechanical Engineering Department, Gebze Turkey

(ercan.erturk@eng.bau.edu.tr, s.selim@gtu.edu.tr, fcakmakbolat@gmail.com)

†Corresponding Author; ercan.erturk@eng.bau.edu.tr, Tel: +90 212 381 0866

Received: 09.07.2018 Accepted: 16.08.2018

Abstract- Dual rotor wind turbines have higher efficiency and they can harvest more energy compared to single rotor wind turbines. In this study a counter-rotating dual rotor wind turbine with double rotational generator armature is modelled with mechanical, aerodynamical and electrical components. The considered model includes permanent magnet synchronous generator (PMSG) and a drive train which represent the mechanical characteristics of the generator. In a dual rotor wind turbine the front rotor extracts some portion of energy from the wind therefore downstream of the front rotor the speed of the air is decreased. The air with reduced speed downstream of the front rotor acts as an inflow for the rear rotor. The speed of the residual wind downstream of the front rotor affects the power output of the dual rotor wind turbine. The resulting torque, aerodynamic power and also electrical power of the counter-rotating dual rotor wind turbine are simulated using MATLAB/Simulink software.

Keywords Dual rotor wind turbine, counter-rotating wind turbine, wind energy, permanent magnet synchronous generator, double rotational armature generator

1. Introduction

Wind is clean, renewable, inexhaustible energy source for electric power generation. The extracted energy from a wind turbine depends on the wind speed, therefore it is economically more profitable to install the wind turbines at regions where the wind power density is high. The extracted energy from a wind turbine also depends on the efficiency of the wind turbine. Today with the use of better design and analysis softwares and also with the use of better manufacturing techniques, most of the wind turbines have efficiencies more or less close to each other with very small differences. If we can increase the efficiency of wind turbines furthermore not only we would be able to extract more energy from the wind at regions with high power density but also it would be economically feasible to install wind turbines at regions with less wind power densities.

Wind turbines that have two rotors, one at the front and one at the rear of the wind turbine, offers high efficiency. A single rotor wind turbine theoretically can extract maximum 59.3% of the energy available in the wind [1] and this limit is known as the Betz limit. Using multiple actuator-disk theory one can easily show that a dual rotor wind turbine theoretically can extract maximum 64% of the energy available in the wind [2]. Therefore the maximum energy that a dual rotor wind turbine can theoretically extract from the wind is 8% higher than that of a single rotor wind turbine.

In the literature it is possible to find many numerical and experimental studies that shows the efficiency of dual rotor wind turbines. Among these studies, Shen et al. [3] have simulated a Nordtank 500 kW wind turbine using Actuator Line technique implemented EllipSys3D Navier-Stokes solver. They [3] have considered the Nordtank 500 kW wind turbine both having a single rotor and also having a counter rotating dual rotor. Using the real wind data measurements taken on the island of Spragø in Denmark, they have concluded that a dual rotor wind turbine can produce 43.5% more energy annually compared to a single rotor wind turbine.
Habash et al. [4] and Mitulet et al. [5] have performed wind tunnel experiments on dual rotor wind turbines. Their [4,5] results indicate that a dual rotor wind turbine can produce 60% more energy than a single rotor wind turbine.

In their study Jung et al. [6] have designed, built and tested a 30 kW dual rotor wind turbine. In this wind turbine, the motion of both rotors are combined with a gear system with bevel gears and planet gears and transmitted perpendicular to an electric generator. After the actual field tests, they [6] stated that the power coefficient of this dual rotor wind turbine is around 0.5, which is well above the power coefficients of single rotor wind turbines.

These different type of studies, numerical [3], experimental [4,5] and field studies [6] all reported that dual-rotor wind turbines have efficiencies around 45-50% which is much greater than that of single rotor wind turbines.

In a dual rotor wind turbine the two rotors can rotate either in co-direction (rotors are rotating in the same direction) or in counter-direction (rotors are rotating in opposite directions). In an experimental study using PIV measurements Ozbay et al. [7] have compared the single rotor wind turbines with dual rotor wind turbines having two rotors rotating in co- and counter-directions in order to investigate the effect of the direction of the rotation of the two rotors on the power extracted from the wind. Ozbay et al. [7] state that a co-rotating dual rotor wind turbine can generate 48% more power compared to a single rotor wind turbine. The increase in the generated power when two rotors are used is remarkable. More importantly Ozbay et al. [7] also state that a counter-rotating dual rotor wind turbine can generate 60% more power compared to a single rotor wind turbine. The study of Ozbay et al. [7] is important in two points; one point is that it shows that dual rotor wind turbines are superior compared to single rotor wind turbines, the second point is that among dual rotor wind turbines counter-rotating dual rotor wind turbines are superior compared to co-rotating dual rotor wind turbines.

Counter rotating dual rotor wind turbines in general can have different mechanical configurations. For example one approach is to combine the counter rotation of both rotors using a gear system and obtain a single rotational output shaft which is connected to the shaft of a generator as it was done in Jung et al. [6]. In this approach, any generator can be used in the wind turbine without any modification since in this approach there is one mechanical rotating shaft output from the gear system as studied in [6]. We note that, in this approach there will be loss of power due to friction in the gear system. Another approach is to use a special generator with double rotational armature as described in [8]. In this approach both the PM rotor and winding stator of the generator can rotate independently, therefore one rotor of the wind turbine is connected to the PM rotor of the generator and the second rotor of the wind turbine is connected to the winding stator of the generator. Since both rotors of the wind turbine rotate in counter directions, i.e. the PM rotor and winding stator armature rotate in counter directions, this approach increases the relative speed of rotation between the PM rotor and winding stator in the generator. We believe that among different approaches of counter rotating dual rotor wind turbines, since it increases the relative speed of rotation in the generator, this approach is superior to the other approaches in terms of efficiency and production of electricity.

In order to maximize the efficiency of the wind turbines while minimizing the loads, wind turbines are operated under control. Since in nature the wind has continuously changing characteristics with continuously varying wind speeds, the aerodynamic torque and/or power of the wind turbine is controlled by changing the pitch angle of the wind turbine blades. Control of wind turbine blades also becomes very important at high wind speeds in a variable speed wind turbine [9]. Linear, nonlinear and intelligent control methods for various wind turbine systems have been studied by many researchers [10,11,12].

In the literature, No et al. [13] and Farahani et al. [14] studied modeling of counter rotating dual rotor wind turbines. In the studies of No et al. [13] and Farahani et al. [14] the counter motion of the wind turbine rotors are combined into a single rotational shaft motion by set of gears which then rotates the generator shaft as the same with that of Jung et al. [6]. The mechanical design, the physics and also modeling of counter rotating dual rotor wind turbine with double rotational generator armature is completely different than that of dual rotor wind turbine considered in No et al. [13] and Farahani et al. [14]. To the best of our knowledge, in the literature there is no study that attempts to model and simulate dual rotor wind turbines with double rotational generator armature and this constitutes the main motivation of this study. In order to reliably predict and analyze the performance of dual rotor wind turbine with double rotational generator armature it is important to have an accurate and reliable model.

In a dual rotor wind turbine, the wind first passes over the front rotor blades and loses some portion of its energy. Then downstream of the front rotor, the residual wind with lower energy passes over the rear rotor blades and loses some portion of its energy again. As one can imagine, since not only the aerodynamics but also the whole system of dual rotor wind turbines are much more complex compared to single rotor wind turbines. In their analysis No et al. [13] and Farahani et al. [14] assumed that the flow is uniform downstream of the front rotor and upstream of the rear rotor, i.e. between the two rotors. Most probably this assumption introduces some errors in the aerodynamic part of their simulation model. However for dual rotor wind turbines since there is no better aerodynamic approximation in the literature for the flow between the two rotors of the dual rotor wind turbine, in this study, following No et al. [13] and Farahani et al. [14] we will also assume that the flow approaching to the rear rotor is uniform.

We note that the rotational speed of the front rotor and the pitching angle of the front rotor blades affect the speed of the flow downstream of the front rotor. The rotational speed of the front rotor and the pitching angle of the front rotor blades can change dynamically during operation of the wind turbine depending on the incoming wind speed and the wind turbine control algorithm. Therefore the wind speed approaching the rear rotor varies during operation of dual turbulence.
rotor wind turbine. In this study, we will examine the variation of the power output of the rear rotor, i.e. therefore the total power output of the dual rotor wind turbine, as a function of the flow speed downstream of the front rotor. For this we will assume different deceleration of the wind after the front rotor and calculate the total power output of the dual rotor wind turbine and obtain the variation of the total power as a function of deceleration. The power output of the rear rotor, i.e. therefore the total power output of the dual rotor wind turbine, is also a function of the rear rotor blade pitch angle. In this study we will also examine the effect of the rear rotor blade pitch angle on the total power output by using various pitch angles for the rear rotor blades.

The aim of this study is then to derive a mathematical model for a counter-rotating dual rotor wind turbine with double rotational generator armature in MATLAB/Simulink environment. Using this mathematical model, the power output of the wind turbine is investigated for different pitch angles of the rear rotor blades in a dual rotor wind turbine. Different case studies are realized in order to analyze the total power output of the dual rotor turbine model for different deceleration values of the wind downstream of the first rotor.

2. Derivation of Mechanical Model

The structure of the considered double rotational generator armature counter-rotating dual rotor wind turbine is schematically shown in Figure 1 as a cross section view. As seen in this figure, the turbine has two counter-rotating rotors, one connected to the stator and one connected to the PM rotor. The rotational speeds of the rotor 1 and rotor 2 are \( \omega_1 \) and \( \omega_2 \), respectively. The moment of inertia of the stator is designated by \( J_1 \) and similarly \( J_2 \) designates the moment of inertia of the PM rotor. Also \( b_1 \) and \( b_2 \) designate the damping in the bearings used in the wind turbine.

In the derivation of the mechanical model, it is assumed that wind blows from right to left as shown in Figure 1 such that the front rotor designated as rotor 1 receives the undisturbed freestream wind and the rear rotor designated as rotor 2 receives the residual decelerated wind. The input and output torques are considered in order to obtain the mechanical model. Here, \( T_1 \) is the input wind torque received by the front rotor. In the turbine side, \( T_{11} \) shows the output mechanical torque generated in the front rotor. Similarly, \( T_2 \) represents the residual decelerated wind torque received by rotor 2 and \( T_{22} \) is the output mechanical torque existed in the rear rotor.

The mechanical dynamics of the dual rotor wind turbine system can be characterized in the most general form by the following equations [15,16].

\[
T_{11} - T_1 - T_{f1} = f_1 \omega_1 + b_1 \omega_1 + k_1 \phi_1 \tag{1}
\]

\[
T_2 - T_{22} = f_2 \omega_2 + b_2 \omega_2 + k_2 \phi_2 \tag{2}
\]

where \( \phi_1 \) and \( \phi_2 \) represent angular displacement of the front and rear rotors, respectively. The relation between angular displacement and rotational speed is \( \omega_1 = \frac{d\phi_1}{dt} \) and \( \omega_2 = \frac{d\phi_2}{dt} \). The rotors rotate in the opposite direction and the direction of \( \omega_1 \) is assumed to be counterclockwise. In equations (1) and (2), \( T_{f1} \) and \( T_{f2} \) denotes the torques due to mechanical friction generated at the bearings of the front and rear rotor respectively. They basically denote the mechanical losses due to friction and their values are calculated explicitly. In matrix form, the system equations are written as

\[
\begin{bmatrix}
J_1 & 0 \\
0 & J_2
\end{bmatrix}
\begin{bmatrix}
\dot{\phi}_1 \\
\dot{\phi}_2
\end{bmatrix}
+ \begin{bmatrix}
b_1 & 0 \\
0 & b_2
\end{bmatrix}
\begin{bmatrix}
\phi_1 \\
\phi_2
\end{bmatrix}
+ \begin{bmatrix}
k_1 & 0 \\
0 & k_2
\end{bmatrix}
\begin{bmatrix}
\phi_1 \\
\phi_2
\end{bmatrix}
= \begin{bmatrix}
T_{11} - T_1 - T_{f1} \\
T_2 - T_{22} - T_{f2}
\end{bmatrix} \tag{3}
\]

Looking at these two equations, at first glance it seems that the front and rear rotor dynamics has no coupling, such that both equations are independent from each other. However the generator torque is actually equal to the sum of the front and rear rotor torques. Therefore in a dual rotor wind turbine with double rotational generator armature as schematically shown in Figure 1, the coupling of the front and rear dynamics appears in the generator torque equation. The relative mechanical angular speed in the generator \( \omega_g \) and the generator torque \( T_e \) are defined as the following

\[
\omega_g = \omega_1 + \omega_2 \\
T_e = T_{11} + T_{22} \tag{4}
\]

Substituting this generator torque into the equation, equation (3) is rearranged as follows

\[
\begin{bmatrix}
J_1 & 0 \\
0 & J_2
\end{bmatrix}
\begin{bmatrix}
\dot{\phi}_1 \\
\dot{\phi}_2
\end{bmatrix}
+ \begin{bmatrix}
b_1 & 0 \\
0 & b_2
\end{bmatrix}
\begin{bmatrix}
\phi_1 \\
\phi_2
\end{bmatrix}
+ \begin{bmatrix}
k_1 & 0 \\
0 & k_2
\end{bmatrix}
\begin{bmatrix}
\phi_1 \\
\phi_2
\end{bmatrix}
= \begin{bmatrix}
\frac{T_e - T_{22} - T_1 - T_{f1}}{T_2 - T_{22} - T_{f2}}
\end{bmatrix} \tag{5}
\]

In this study we have assumed that the damping in the bearings are negligible and also we have assumed that the whole system is a semidefinite system with one degree of freedom.
freedom such that the rotating mechanical parts are rigid. Thus the damping coefficient of both front and rear rotor \((b_1 = b_2 \approx 0)\) and also the stiffness coefficient of both front and rear rotor \((k_1 = k_2 \approx 0)\) are taken as zero. After these assumptions the governing equations for the mechanical model simplifies to the following

\[
\begin{bmatrix}
I_1 & 0 \\
0 & I_2
\end{bmatrix}
\begin{bmatrix}
\ddot{\phi}_1 \\
\ddot{\phi}_2
\end{bmatrix}
= \begin{bmatrix}
T_e - T_{22} - T_1 - T_{f1} \\
T_2 - T_{22} - T_{f2}
\end{bmatrix}
\]

(6)

3. Derivation of the Aerodynamic Model

In a horizontal axis wind turbine, as wind passes over the turbine blade, due to the aerodynamic lift the wind exerts a torque on the turbine blades. In other words the wind loses part of its energy and this energy is converted into mechanical energy. In general, the aerodynamic torque developed on the turbine blades is calculated as \([17,18,19]\)

\[
T = \frac{0.5 \rho \pi R^2 V_s^2 C_p(\beta, \gamma)}{\lambda}
\]

(7)

where \(\rho\) is the air density, \(R\) is the turbine blade radius, \(V_s\) is the incoming free stream wind speed and \(C_p\) is the power coefficient. Also, the tip speed ratio is defined as \(\lambda = \omega R/V_s\), where \(\omega\) is the angular speed of the turbine rotor. The power coefficient \(C_p\) can be written as a function of the blade pitch angle \(\beta\) and \(\gamma\) variable. In this study, for the aerodynamic model the following equations are used \([20,21,22]\)

\[
C_p = 0.22 \left(\frac{116}{\gamma} - 0.4 \beta - 5\right) \exp\left(-\frac{12.5}{\gamma}\right)
\]

\[
\gamma = \frac{1}{\lambda + 0.08 \beta - \frac{0.035}{\beta^3 + 1}}
\]

(8)

In this study in all simulations, we have assumed that the blades of the front rotor have a fixed pitch angle \(\beta = 4^\circ\). For pitch angle of \(\beta = 4^\circ\), the corresponding power coefficient \(C_p\) of the front rotor is given in Figure 2. For the rear rotor we have considered three different pitch angles \(\beta = 4^\circ, 8^\circ, 12^\circ\) and the corresponding power coefficients for the rear rotor for these pitch angles are shown in Figure 3. In a dual rotor wind turbine, since the front rotor blades and rear rotor blades may have different radii and since both rotors have different incoming wind speed and also since both rotors may have different angular speeds, the tip speed ratios of the front and rear rotors are different. Consequently the aerodynamic torque developed on the front and rear rotors are also different. Therefore, the torques of the front and rear rotors are considered separately as follows:

\[
T_1 = \frac{0.5 \rho \pi R_1^2 V_s^2 C_{p1}(\beta_1, \lambda_1)}{\lambda_1}
\]

\[
T_2 = \frac{0.5 \rho \pi R_2^2 V_s^2 C_{p2}(\beta_2, \lambda_2)}{\lambda_2}
\]

(9)

From these, the aerodynamic powers of the front and rear rotors are calculated as the following

\[
P_1 = T_1 \omega_1
\]

\[
P_2 = T_2 \omega_2
\]

(10)

Finally, the total aerodynamic torque and also the total aerodynamic power of the considered dual rotor wind turbine is obtained as the following

\[
T_w = T_1 + T_2
\]

\[
P_w = P_1 + P_2
\]

(11)

4. Derivation of the Electrical Model

In the modeled dual rotor wind turbine with double rotational generator armature, the rotor and stator of the generator rotate in opposite directions as shown in Figure 1. In this type of counter-rotating wind turbine structure, the main idea is to increase the relative speed in the generator. The considered wind turbine generator is a direct drive generator without any gear box, neither on the front nor on the rear rotors. Also the generator of the wind turbine is a permanent magnet synchronous generator (PMSG) and the electrical model is constructed accordingly.

The dynamic model of PMSG is derived from the two-phase synchronous reference frame in which the q-axis is 90° ahead of the d-axis with respect to the direction of the rotation. The equations are given as \([23,24,25]\)

\[
\frac{di_d}{dt} = -\frac{R_a}{L_d} i_d + \omega_e \frac{L_q}{L_d} i_q + \frac{1}{L_d} u_d
\]

(12)

\[
\frac{di_q}{dt} = -\frac{R_a}{L_q} i_q - \omega_e \left(\frac{L_d}{L_q} i_d + \frac{1}{L_q} \psi_q\right) + \frac{1}{L_q} u_q
\]

(13)

where \(u_d, u_q, i_d, i_q\) are component of the stator voltages and currents. Also, \(R_a\) is armature resistance, \(\psi_q\) is magnetic flux, \(L_d\) and \(L_q\) are inductances of the generator at q and d
axes. Assuming that $L = L_d = L_q$ the equations (12) and (13) become

$$\frac{di_d}{dt} = -\frac{R_a}{L}i_d + \omega_e i_q + \frac{1}{L}u_d$$

$$\frac{di_q}{dt} = -\frac{R_a}{L}i_q - \omega_e \left( i_q + \frac{1}{L}\psi_0 + \frac{1}{L}u_q \right)$$

(14)

(15)

In general, the electromagnetic torque is defined as

$$T_e = 1.5 n_p i_q \left( (L_d - L_q)i_q + \psi_0 \right)$$

(16)

Since the inductance of the generator is accepted as $L = L_d = L_q$, then the electromagnetic torque becomes

$$T_e = 1.5 n_p i_q \psi_0$$

(17)

where $n_p$ represents the pole pairs. The electrical rotating speed is defined as

$$\omega_e = n_p \omega_q$$

(18)

In our calculations instead of using the total electrical rotating speed ($\omega_e$) defined above, similar to the calculations we have done in aerodynamic model, we assumed that the electrical rotating speed ($\omega_e$) is composed of two parts and we introduced two new variables as the following

$$\omega_{e1} = n_p \omega_1$$

$$\omega_{e2} = n_p \omega_2$$

(19)

Here $\omega_{e1}$ is the electrical rotating speed due to the rotation of the front rotor and similarly $\omega_{e2}$ is electrical rotating speed due to the rotation of the rear rotor. The relation between the electrical rotating speed ($\omega_e$) and the new introduced variables $\omega_{e1}$ and $\omega_{e2}$ is defined as the following

$$\omega_e = \omega_{e1} + \omega_{e2}$$

(20)

Instead of calculating the total electrical rotating speed ($\omega_e$) with one unknown, we find it more convenient and consistent with the calculations done in aerodynamic model and calculate the value using two new parameters $\omega_{e1}$ and $\omega_{e2}$. Mathematically it makes no difference except that we are solving for one more unknown. Using these introduced new variables, just like the same we do in the aerodynamic model we define the electrical powers due to the motion of the front and rear rotors as the following

$$P_{e1} = \frac{T_{e1} \omega_{e1}}{n_p}$$

$$P_{e2} = \frac{T_{e2} \omega_{e2}}{n_p}$$

(21)

Finally the generator power (total electrical power) $P_e$ is calculated as follows

$$P_e = P_{e1} + P_{e2}$$

(22)

Since we have introduced two new variables $\omega_{e1}$ and $\omega_{e2}$, we solve the equations (14) and (15) twice, once for the front rotor in order to find $\omega_{e1}$ and once for the rear rotor in order to find $\omega_{e2}$. After $\omega_{e1}$ and $\omega_{e2}$ is obtained, the electromagnetic torque and the generator power is calculated easily.

5. Simulation Results and Discussions

5.1. Simulation of Test Scenario for Validation

We have created a MATLAB/Simulink model a for the considered dual rotor wind turbine. In order to validate our MATLAB/Simulink model we first decided to run a known simulation scenario to see if the model produces the expected results. For this test we have created a MATLAB/Simulink file shown in Figure 4. In this test we have used the parameters given in Table 1 and also the pitch angle of the rear rotor is taken as the same with that of the front rotor such that the blade pitch angle of both front and rear rotor is $\beta = 4^\circ$. In this test simulation we have assumed that there are no friction losses in the system such that both $T_{f1}$ and $T_{f2}$ are equal to zero. Moreover we also have assumed that both the front rotor and the rear rotor have the same freestream wind speed such that

$$V_{sr} = V_s$$

(23)

We note that when the wind passes over the front rotor it loses part of its energy and therefore slows down. Thus the wind speed upstream of the rear rotor is lower than the wind speed upstream of the front rotor in reality. With the assumption given in equation (23) we are assuming that the wind still has the same speed even after passing the front rotor. Although it is physically unrealistic, the assumption that both the front rotor and the rear rotor have the same

![Fig. 4. Matlab/Simulink model of the dual rotor wind turbine for the validation test case](image-url)

**Table 1. Parameters used in the model for the validation test case**

<table>
<thead>
<tr>
<th>No</th>
<th>Item</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Radius of wind turbine rotor 1</td>
<td>$R_1$</td>
<td>2.5</td>
<td>m</td>
</tr>
<tr>
<td>2</td>
<td>Radius of wind turbine rotor 2</td>
<td>$R_2$</td>
<td>2.5</td>
<td>m</td>
</tr>
<tr>
<td>3</td>
<td>Moment of inertia of the front rotor</td>
<td>$J_1$</td>
<td>18.138</td>
<td>kg(\cdot)m$^2$</td>
</tr>
<tr>
<td>4</td>
<td>Moment of inertia of the rear rotor</td>
<td>$J_2$</td>
<td>18.138</td>
<td>kg(\cdot)m$^2$</td>
</tr>
<tr>
<td>5</td>
<td>Mechanical friction loss at the front rotor</td>
<td>$T_{fr}$</td>
<td>0</td>
<td>N(\cdot)m</td>
</tr>
<tr>
<td>6</td>
<td>Mechanical friction loss at the rear rotor</td>
<td>$T_{fr}$</td>
<td>0</td>
<td>N(\cdot)m</td>
</tr>
<tr>
<td>7</td>
<td>Magnetic flux</td>
<td>$\psi_b$</td>
<td>0.76</td>
<td>Wb</td>
</tr>
<tr>
<td>8</td>
<td>Resistance</td>
<td>$R_s$</td>
<td>1.5</td>
<td>Ω(\cdot)m</td>
</tr>
<tr>
<td>9</td>
<td>Inductance</td>
<td>$L$</td>
<td>14.04</td>
<td>mH</td>
</tr>
<tr>
<td>10</td>
<td>Number of pole pairs</td>
<td>$n_p$</td>
<td>8</td>
<td></td>
</tr>
</tbody>
</table>
freestream wind speed offers a perfect mathematical test case for the MATLAB/Simulink model we developed. In a counter-rotating dual rotor wind turbine, if all of the mechanical losses are assumed to be zero then the following statements must be satisfied in this test simulation

a) Since the radius of both front rotor and the rear rotor are taken as the same, the blade pitch angle of the front and rear rotor are also the same and also the wind speed upstream of both the front and rear rotor are the same, the aerodynamic power generated both at the front rotor and also at the rear rotor should be the same.

b) Since we have assumed that there are no losses in the system then the total aerodynamic power generated by the two rotors should be equal to the electrical power generated at the generator.

c) Similarly for the same reason the total aerodynamic torque should be equal to the electromagnetic torque.

In this test simulation we have assumed that the wind speed is constant and equal to 10m/s and there is no variation in wind speed in time. The considered wind speed profile is given in Figure 5. The calculated aerodynamic and electrical torques for the front and rear rotors and together with the total aerodynamic torque and the electromagnetic torque is given in Figure 6. As it is expected, the electromagnetic torque asymptotes to a value in time which is equal to the total aerodynamic torque in absolute sense. Figure 7 shows the aerodynamic power of the front rotor and rear rotor and also the total combined power. As it is expected, since the radius of both front and rear rotors are the same, and also the incoming wind speed to both rotors are assumed to be the same, the power of the front rotor and the rear rotor are the same at the steady state. In Figure 8 the total aerodynamic power and the generator power are plotted, again as expected since the system is assumed to have no mechanical losses, both aerodynamic power and the generator power asymptotes to the same value at the steady state. This simulation test proves that our Matlab/Simulink model indeed produces correct results.

In the results presented above the blade pitch angle of both the front and the rear rotor was taken as \( \beta = 4^\circ \). Since the difference between a single rotor and a dual rotor wind turbine is the additional second rear rotor at the back, we decided to see how the blade pitch angle of this additional rear rotor pitch angle

![Fig. 5. Wind speed profile for the front and rear rotor](image)

![Fig. 6. Aerodynamic and electromagnetic torques of the dual rotor turbine model](image)

![Fig. 7. The aerodynamic power of the two rotors of the dual rotor wind turbine](image)

![Fig. 8. Power outputs of the dual rotor turbine model](image)

![Fig. 9. Variation of the rotational speed of the rear rotor with rear rotor pitch angle](image)
In order to analyze and simulate the rear rotor we need to specify the residual wind speed downstream of the front rotor that is to say we need to specify the wind speed upstream of the rear rotor. Following No et al. [13] and Farahani et al. [14] we will assume that the flow approaching to the rear rotor is uniform. According to the Betz law the wind speed after the front rotor should be greater than 0.5 of the incoming wind speed. Therefore in this study we will assume different values for the residual wind speed incoming to the rear rotor after the front rotor and run the simulation for these different deceleration values in order to see the effect of the residual wind speed on the power output of the simulated dual rotor wind turbine.

In the previous test scenario since the aim was only to test our Matlab/Simulink model of the dual rotor wind turbine, we have considered an unrealistic steady constant wind speed given in Figure 5. In reality wind speed continuously varies in time with fluctuations due to turbulence. For a more realistic simulation we have considered the distributed white noise base wind model given in [26]. Using the distributed white noise base wind model we have generated a wind profile \( V_f \) for the freestream wind speed incoming to the front rotor as given in Figure 11. In generating this wind profile, in the distributed white noise base wind model the average wind speed is taken as 10 m/s and the rate of turbulence is taken as 4%. The incoming wind speed of the rear rotor \( V_{fr} \) is taken as 0.7 of the incoming wind speed of the front rotor such that there is a 30% wind speed loss (deceleration) after the front rotor.

\[
V_{fr} = 0.7 V_f
\]  

![Fig. 10. Variation of the total power of the dual rotor wind turbine with rear rotor pitch angle](image)

5.2. Simulation of Dual Rotor Wind Turbine

According to the Betz Law, an ideal wind turbine will decelerate the wind by 2/3 of its original speed. In other words the wind speed at far away downstream of an ideal wind turbine is equal to 1/3 of the freestream wind speed at far away upstream of the wind turbine. Also for an ideal wind turbine, the wind speed at the plane of the rotor is equal to the 2/3 of the freestream wind speed at far away upstream of the wind turbine, thus the wind speed right after the rotor is equal to 0.67 of the incoming freestream wind speed. Also the Betz analysis is valid only for deceleration values of 50% or above at the plane of the rotor. In other words for the Betz analysis to be valid the wind speed right after the rotor can be minimum equal to 0.5 of the incoming freestream wind speed. We note that these numbers are valid only for an ideal wind turbine and in real applications the wind does not decelerate to the ideal limit. For most modern wind turbine blades the axial induction coefficient is less than 0.3 along the blade.

![Fig. 11. Wind speed profile obtained by the distributed white noise base wind model](image)

In order to analyze and simulate the rear rotor we need to specify the residual wind speed downstream of the front rotor that is to say we need to specify the wind speed upstream of the rear rotor. Following No et al. [13] and Farahani et al. [14] we will assume that the flow approaching to the rear rotor is uniform. According to the Betz law the wind speed after the front rotor should be greater than 0.5 of the incoming wind speed. Therefore in this study we will assume different values for the residual wind speed incoming to the rear rotor after the front rotor and run the simulation for these different deceleration values in order to see the effect of the residual wind speed on the power output of the simulated dual rotor wind turbine.

In the previous test scenario since the aim was only to test our Matlab/Simulink model of the dual rotor wind turbine, we have considered an unrealistic steady constant wind speed given in Figure 5. In reality wind speed continuously varies in time with fluctuations due to turbulence. For a more realistic simulation we have considered the distributed white noise base wind model given in [26]. Using the distributed white noise base wind model we have generated a wind profile \( V_f \) for the freestream wind speed incoming to the front rotor as given in Figure 11. In generating this wind profile, in the distributed white noise base wind model the average wind speed is taken as 10 m/s and the rate of turbulence is taken as 4%. The incoming wind speed of the rear rotor \( V_{fr} \) is taken as 0.7 of the incoming wind speed of the front rotor such that there is a 30% wind speed loss (deceleration) after the front rotor.

\[
V_{fr} = 0.7 V_f
\]  

![Fig. 12. Matlab/Simulink model of the dual rotor wind turbine](image)

### Table 2. Parameters used in the model for the dual rotor wind turbine simulation

<table>
<thead>
<tr>
<th>No</th>
<th>Item</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Radius of wind turbine rotor 1</td>
<td>( r_1 )</td>
<td>2.5</td>
<td>m</td>
</tr>
<tr>
<td>2</td>
<td>Radius of wind turbine rotor 2</td>
<td>( r_2 )</td>
<td>2.5</td>
<td>m</td>
</tr>
<tr>
<td>3</td>
<td>Moment of inertia of the front rotor</td>
<td>( I_f )</td>
<td>18.138</td>
<td>kg(\cdot)m(^2)</td>
</tr>
<tr>
<td>4</td>
<td>Moment of inertia of the rear rotor</td>
<td>( I_r )</td>
<td>18.138</td>
<td>kg(\cdot)m(^2)</td>
</tr>
<tr>
<td>5</td>
<td>Mechanical friction loss at the front rotor</td>
<td>( \tau_{fr} )</td>
<td>1.8</td>
<td>Nm</td>
</tr>
<tr>
<td>6</td>
<td>Mechanical friction loss at the rear rotor</td>
<td>( \tau_{fr} )</td>
<td>0.9</td>
<td>Nm</td>
</tr>
<tr>
<td>7</td>
<td>Magnetic flux</td>
<td>( \psi_b )</td>
<td>0.76</td>
<td>Wb</td>
</tr>
<tr>
<td>8</td>
<td>Resistance</td>
<td>( R_b )</td>
<td>1.5</td>
<td>( \Omega )</td>
</tr>
<tr>
<td>9</td>
<td>Inductance</td>
<td>( L )</td>
<td>14.04</td>
<td>mH</td>
</tr>
<tr>
<td>10</td>
<td>Number of pole pairs</td>
<td>( n_p )</td>
<td>8</td>
<td>-</td>
</tr>
</tbody>
</table>
Figure 11 also shows the wind profile used in our simulation for the rear rotor ($V_{sr}$).

The Matlab/Simulink model of the dual rotor wind turbine is given in Figure 12 and the parameters used in the simulation is listed in Table 2. We note that in this new simulation in order to have more realistic conditions we utilize the system with the frictional losses. In order to calculate the frictional losses we first estimate the weight of the mechanical parts connected to the front rotor and also that of the mechanical parts connected to the rear rotor. As seen in Figure 1, since the weight of the mechanical parts connected to the front rotor (the weight of the stator) is larger than the weight of the mechanical parts connected to the rear rotor (the weight of the PM rotor) we have assumed that there is a 2:1 ratio in the weights of the front and rear parts.

Using the estimated weights we have calculated the equivalent dynamic bearing loads and from this we have calculated the frictional moment in the bearings. The estimated values of the mechanical friction losses are given in Table 2.

The obtained electromagnetic torques, the aerodynamic torques, the angular speeds and the generator powers are given in Figure 13, 14, 15 and 16 respectively.

We note that Figure 13, 14, 15 and 16 shows the results for 30% loss of wind speed at the rear rotor side of the dual rotor wind turbine and also for blade pitch angle of $\beta = 4^\circ$ both at the front and rear rotor. The aim of using a dual rotor structure in a wind turbine is to maximize the energy capture from the wind. The power output of the dual rotor wind turbine depends both on the loss in the wind speed after the front rotor and also the pitch angle of the blades. We have repeated the simulation considering different losses in the wind speed after the front rotor. In these simulations we have also considered different pitch angles for the rear rotor and use $\beta = 4^\circ, 8^\circ, 12^\circ$ for the rear rotor. Table 3 tabulates the time averaged power values for different values of the loss in the wind speed after the front rotor and also for different values of the pitch angle of the rear rotor blades.

In a dual rotor wind turbine the rear rotor captures some power additional to the front rotor and this amount of power can be considered as the benefit of using a second rotor in a wind turbine. In order to quantify this benefit we have defined a power increase rate as the following

$$R = \left( \frac{P_{e2}}{P_{e1}} - 1 \right) \times 100 \quad (25)$$

The power increase rate can also be written as the following

$$R = \left( \frac{P_{e2}}{P_{e1}} \right) \times 100 \quad (26)$$

<table>
<thead>
<tr>
<th>Rear side wind speed loss (%)</th>
<th>Rear rotor Electromagnetic power [Watt]</th>
<th>Rear rotor generator power [Watt]</th>
<th>Dual rotor generator power</th>
<th>Dual rotor turbine power increase rate [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta = 4 )</td>
<td>( \beta = 8 )</td>
<td>( \beta = 12 )</td>
<td>( \beta = 4 )</td>
<td>( \beta = 8 )</td>
</tr>
<tr>
<td>0%</td>
<td>5949</td>
<td>3730</td>
<td>1913</td>
<td>11817</td>
</tr>
<tr>
<td>10%</td>
<td>3724</td>
<td>2376</td>
<td>1238</td>
<td>9587</td>
</tr>
<tr>
<td>20%</td>
<td>2251</td>
<td>1456</td>
<td>765</td>
<td>8118</td>
</tr>
<tr>
<td>30%</td>
<td>1299</td>
<td>845</td>
<td>445</td>
<td>7160</td>
</tr>
<tr>
<td>40%</td>
<td>688</td>
<td>451</td>
<td>239</td>
<td>6555</td>
</tr>
<tr>
<td>50%</td>
<td>331</td>
<td>216</td>
<td>115</td>
<td>6198</td>
</tr>
</tbody>
</table>

Fig. 13. Electromagnetic torques of the dual rotor

Fig. 14. Aerodynamic torques of the dual rotor

Fig. 15. The angular speed of the front rotor and rear rotors

Fig. 16. Generator output powers of the dual rotor
This defined power increase rate basically shows the increase in the power output of a single rotor wind turbine when a second rotor is added to the rear side of the wind turbine additional to the front rotor. We note that in Table 3 in all of the calculations the average power output of the front rotor is equal to $P_{e1} = 5949$ Watt and it does not change when the amount of loss in the wind speed downstream of the front rotor and also the pitch angle of the rear rotor changes.

Using the values given in Table 3, Figure 17 shows the variation in power increase rate ($R$) as a function of loss in the wind speed after the front rotor for different pitch angles of the rear rotor blades.

![Fig. 17. Relation between loss in residual wind and power output](image)

6. Conclusions

In this study mechanical and electrical model of a small scale counter-rotating dual rotor wind turbine is derived and the increase in the power output is investigated for different cases. The simulation results are obtained in MATLAB/Simulink environment. The power increase of the dual rotor turbine strongly depends on residual wind speed coming onto the rear rotor of the wind turbine. In addition, pitch angle of the blades have important role on the produced total power as well.

References


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