# Distributed Generations Optimal Placement and Sizing in Unbalanced Distribution Systems with Respect to Uncertainties

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Abstract- Distributed generations are renewable sources that do not create environmental pollution and install near the site of consumption. Also, the generated power of wind and solar sources are faced with uncertainty due to the dependence on external factors. With determination of the appropriate location and size of these sources in distribution networks their positive effects can be realized. But it must be noted that distribution networks due to the unequal phase loads and mutual impedance between various phases of lines are virtually unbalanced. So in this paper, determination of the appropriate size and place of distributed generations is done in unbalanced distribution network that distributed generations' appropriate size obtained with respect to probabilistic methods and their convenient location is selected using weighted multi-objective IPSO algorithm. With regard to impact of exact load flow in calculations, linear three-phase unbalanced load flow method considering the loads model and loads connection type is used which is the fast and accurate unbalanced load flow method. Simulations are done in IEEE 37 bus unbalanced network in Matlab software. The simulation results indicate that network power losses and voltage unbalance factor are reduced, voltage profile of each phase is improved and significant profit is obtained for distribution companies.

**Keywords** Improved Particle Swarm Optimization, Linear Three Phase Unbalanced Load Flow, Unbalanced Distribution Systems, Uncertainty of Distributed Generations.

## 1. Introduction

Nowadays, power system operators tend to generate power in distributed voltage level due to increased demand for electrical energy. For this purpose, distributed generation sources are used in these networks. In terms of IEEE, distributed generations are electric power generation sources that they are not directly connected to the global transmission system and include generators and saving energy technologies [1]. In general, the advantages of distributed generation sources can be classified into two categories; technical and economic. The technical advantages of distributed generation sources include reducing of network power losses, improving of voltage profile, reducing of environmental pollution, improving of power quality, reliability and network security and increasing of energy efficiency. [2] Their economic advantages include different investments to improve facilities, optimal generation, reducing of operation costs and etc. [3]. Currently, wind and solar distributed generation sources are used more. The amount of output power of these units due to variable wind speed and solar radiation during the day is not constant. So the output power of these units is uncertain [4]. Uncertainty of imprecisely parameters is considered using fuzzy systems and probabilistic methods. Contingency or randomize planning convert random and uncertainty variables to common linear and nonlinear programming.

Distribution networks are actually unbalanced. It can be noted that the main reasons of distribution systems' imbalance are different loading of single-phase loads on three-phase loads and mutual impedance between different

phases of lines [5]. Existence of imbalance in the distribution networks have considerable negative effects, such as increasing of power losses, voltage drop, occupying network capacity and increasing of costs [6]. Due to the specific characteristics of unbalanced distribution networks and high R/X ratio, using transmission networks load flow methods such as Newton-Raphson and Gauss Seidel are not suitable for unbalanced distribution networks [7]. In usual methods of load flow, loads power are be considered constant and type of loads connection have been ignored, while in distribution network, loads are dependent on the voltage and loads connection can be as delta or star. Several studies have been done on the above mentioned topics. In [8] a three-phase power flow solution method using graph theory, injection current, and sparse matrix techniques for large-scale unbalanced distribution networks and in the bus frame of reference, a direct iterative method is adopted. In [9] a genetic algorithm based technique for the optimal allocation of DG<sup>†</sup> units in the power systems has been proposed. In [10] a method of solving three phase probabilistic load flow for distribution system with single-phase and three-phase gridconnected photovoltaic systems using the sequential Monte Carlo technique has been presented and time-sequence impacts of PV generation on power flow and voltage profiles in distribution system, taking into account the correlation of weather between different zones, are gained from results. In [11] an efficient approach to feeder reconfiguration for power loss reduction and voltage profile improvement in unbalanced radial distribution systems has been presented and forward- backward algorithm is used to calculate load flow.

In this paper, determination of the location and size of DGs is considered in unbalanced distribution network. Hence, three-phase unbalanced load flow calculations have been done by linear three phase unbalanced method considering loads model and connection type. First size of DGs based on the uncertainty methods is obtained and then by weighted multi-objective IPSO algorithm the exact location of sources is selected. Simulations have been implemented in Matlab software on IEEE 37 bus unbalanced network that has unequal loads and asymmetric structure of lines. The simulation results show that voltage profile of each phase is improved, network power loss and voltage unbalance factor are reduced and great benefit is obtained for distribution companies.

## 2. Multi Objective Function

The multi objective function in this paper include the reduction of network active power losses, voltage profile improvement and reduction of the voltage unbalance factor. Then the profits from the installation of the DGs obtain.

## 2.1. Reducing of Network Active Power Losses

One of the important parameters of network is reduce losses and it is possible with the help of the DGs. power losses of unbalanced distribution network is obtained as follows [12]:

$$F_{1} = P_{Loss} = \sum_{P=A,B,C} P_{Loss}^{(P)}$$

$$= \sum_{P=A,B,C} \left[ \sum_{i=1}^{n} \sum_{j=1}^{n} V_{i}^{(P)} Y_{ij}^{(P)} V_{j}^{(P)} \cos(\theta_{i}^{(P)} - \theta_{j}^{(P)} - \delta_{ij}^{(P)}) \right]$$
(1)

Where

- P Phases A, B and C;
- n Number of network buses;
- $V_i^{(P)}$  Voltage size of phase P at bus i;
- $\theta_i^{(P)}$  Voltage angle of phase P at bus i;
- $V_{i}^{(P)}$  Voltage size of phase P at bus j;
- $\theta_{i}^{(P)}$  Voltage angle of phase P at bus j;
- $Y_{ii}^{(P)}$  Admittance size of phase P between i and j buses;
- $\delta_{ii}^{(P)}$  Admittance angle of phase P between i and j buses.

## 2.2. Improving of Voltage Profile

Optimal DG placement can improve voltage profile of phases. In fact, the purpose of improving the voltage profile is that the voltage of each phase is close to 1 perunit [13].

$$VD_{A} = |V_{i,ref} - V_{i,A}|$$

$$VD_{B} = |V_{i,ref} - V_{i,B}|$$

$$VD_{C} = |V_{i,ref} - V_{i,C}|$$
(2)

That  $V_{i,A}$ ,  $V_{i,B}$  and  $V_{i,C}$  are voltage size of bus i in phases A, B and C respectively, and  $V_{i,ref}$  is the desired voltage value, which is usually considered 1 perunit. The aim of this section is that Eq. (3) is minimized.

$$F_2 = \max(VD_A, VD_B, VD_C)$$
(3)

## 2.3. Reducing of Voltage Unbalance Factor

According to IEEE standard, VUF<sup>‡</sup> is obtained as follows [14]:

$$\% VUF_{i} = \frac{V_{2,i}}{V_{1,i}} \times 100 \tag{4}$$

That  $V_{2,i}$  and  $V_{1,i}$  are respectively voltage components of positive and negative of bus i which obtained based on symmetrical component method as Eq. (5).

$$\begin{bmatrix} V_{0} \\ V_{1} \\ V_{2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^{2} \\ 1 & a^{2} & a \end{bmatrix} \begin{bmatrix} V_{a} \\ V_{b} \\ V_{c} \end{bmatrix}$$
(5)

That  $V_{_{a}}\,,\,\,V_{_{b}}\,$  and  $\,V_{_{c}}\,$  are respectively phase voltages of phase

<sup>&</sup>lt;sup>+</sup> Distributed Generation

<sup>&</sup>lt;sup>‡</sup> Voltage Unbalance Factor

a, b and c and  $V_0$ ,  $V_1$  and  $V_2$  are respectively voltage components of zero, positive and negative and  $a = e^{j120}$ .

The aim of this section is that Eq. (6) is minimized.

$$F_3 = \max(VUF_i) \tag{6}$$

To improve efficiency and achieve better results, the objective function is defined as a combination of the three above objectives and weighted sum of these objectives is considered as Eq. (7).

$$F = k_1 \times F_1 + k_2 \times F_2 + k_3 \times F_3 \tag{7}$$

 $k_1 + k_2 + k_3 = 1$  ,  $k_1, k_2, k_3 \in [0,1]$ 

Values of the  $k_1, k_2, k_3$  coefficients are selected based on the importance of objectives in such a way that the best results are achieved. It should be noted that each of the objectives unit is based on perunit.

### 3. Network Constraints

Network constraints are defined as follows:

- Voltage of each phase, should be between the standard limit that is considered to be 0.9-1.1 (perunit) or close to it [15].
- Maximum value of VUF<sub>i</sub> must be less than allowable voltage unbalance that is considered 2 percent [14].
- Network losses with presence of DG must be less than the losses without DG.

### 4. Linear Three Phase Unbalanced Load Flow

Because of unequal phase loads and asymmetric structure of lines, load flow methods are different between unbalanced distribution networks and balanced distribution networks. In this paper, linear three-phase unbalanced load flow is used. In this method, loads model and connection type are considered. So compared with other methods of load flow, it is faster and more accurate [16]. Buses voltage and current are connected by admittance matrix as follows:

$$\begin{pmatrix} I_{s} \\ I_{N} \end{pmatrix} = \begin{pmatrix} Y_{ss} & Y_{sN} \\ Y_{Ns} & Y_{NN} \end{pmatrix} \cdot \begin{pmatrix} V_{s} \\ V_{N} \end{pmatrix}$$
(8)

That diagonal elements represent insider admittance and non-diagonal elements represent mutual admittance,  $I_s$  and  $V_s$  are respectively current and voltage of bus S (Slack bus),  $I_N$  and  $V_N$  are respectively current and voltage of bus with N index (other buses). The N index includes three phases so size of admittance matrix is  $(N_{bus} \times 3) \times (N_{bus} \times 3)$  that  $N_{bus}$  represents the number of network buses.

Load model of bus k is shown by the relationship between voltage and current of its as following equation that  $h = 1/V_{nom}$  and  $V_{nom}$  is network nominal voltage.

$$I_{k} = \frac{S_{Pk}^{*}}{V_{k}^{*}} + h \cdot S_{Ik}^{*} + h^{2} \cdot S_{Zk}^{*} \cdot V_{k}$$
(9)

Every part of this equation represents one type of load model current as follow:

- $\succ$  Constant power load model current  $(S_{Pk}^*/V_k^*)$
- Constant current load model current ( $\mathbf{h} \cdot \mathbf{S}_{ik}^*$ )
- Constant impedance load model current  $(h^2 \cdot S^*_{z_k} \cdot V_k)$

That  $S_{Pk}, S_{Ik}, S_{Zk}$  are respectively network powers in constant power load model, constant current load model and constant impedance load model.

Equation (9) is turn to linear relation by following equation that is obtained from the Taylor series.

$$\frac{1}{V} = \frac{1}{1 - \Delta V} \approx 1 + \Delta V = 2 - V \tag{10}$$

Type of connection load can be as delta or a star. Bus voltage is changed according to the type of connection loads. To convert delta's line voltage  $(V_{N(Line)})$  to star's phase voltage  $(V_{N(Phase)})$  the following equation is used.

$$V_{N(Ime)} = M \cdot V_{N(Phase)} \tag{11}$$

M matrix is defined as follows:

$$M = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}$$
(12)

After linearization of Eq. (9) and using Eq. (11) in it, linear relationship can be arranged as follows:

$$A + BV_{N}^{*} + CV_{N} = 0 (13)$$

The real and imaginary values of voltage (respectively with r and i index) can be obtained as follows:

$$\begin{pmatrix} -A_r \\ -A_i \end{pmatrix} = \begin{pmatrix} B_r + C_r & B_i - C_i \\ B_i + C_i & -B_r + C_r \end{pmatrix} \cdot \begin{pmatrix} V_r \\ V_i \end{pmatrix}$$
(14)

That A, B and C are obtained as follows:

$$A = Y_{NS} \cdot V_{S} - 2h \cdot M^{T} \cdot S_{PN}^{*} \cdot T - h \cdot M^{T} \cdot S_{IN}^{*} \cdot T$$
(15)

$$B = h^{2} \cdot M^{T} \cdot diag \left(S_{PN}^{*} \cdot T^{2}\right) \cdot M$$
(16)

$$C = Y_{NN} - h^2 \cdot M^T \cdot diag(S_{2N}^*) \cdot M$$
(17)

That  $T = e^{j\varphi}$  and  $\varphi = \{0, -2\pi/3, 2\pi/3\}$ .

After obtaining all buses voltage, load flow calculations have ended and network power losses is calculated.

## 5. Uncertainty Modeling of DGs Generated Power

The unavailability of accurate prediction of wind speed and solar radiation, faced the amount of wind turbines and solar panels generated power with uncertainty in the time periods studied. With regard to the wind speed predicted probability distribution and hourly distribution of solar radiation, output power of these units can be obtained due to the uncertainties. In this section, uncertainty modeling of solar and wind power have been investigated.

## 5.1. Uncertainty Modeling of Wind Turbine Generated Power

Wind speed profile in a definite place follows the Weibull distribution function which is obtained as follows [17]:

$$f_{w}(v) = \frac{k}{c} \left(\frac{v}{c}\right)^{k-1} \exp\left[-\left(\frac{v}{c}\right)^{k}\right]$$
(18)

That v is wind speed (m/s), k is shape coefficient and c is scale coefficient. These coefficients are obtained as follows [17]:

$$k = \left(\frac{\sigma}{\mu}\right)^{-1.086} \tag{19}$$

$$c = \frac{\mu}{\Gamma\left(1 + \frac{1}{k}\right)} \tag{20}$$

That  $\mu$  and  $\sigma$  are respectively the average wind speed and standard deviation. To prevent high volume calculations, wind speed changes is divided into interval of 1 (m/s) and the middle of each interval is considered as its representative. So the probability of wind speed for each part can be achieved by the following equation [17].

$$P_{\nu}\{G_{\nu}\} = \int_{\nu_{w2}}^{\nu_{w1}} f_{\nu}(\nu) d\nu$$
(21)

Where  $f_w(v)$  is probability density function of wind speed,  $v_{w1}$  and  $v_{w2}$  are wind speed limits of state w. In wind turbines, output active power was affected by wind speed and a linear relationship between these two parameters are as follows [17]:

$$P_{out}(v) = \begin{cases} 0, & 0 \le v_{aw} \le v_{ci} \\ P_{rated} * \frac{v_{aw} - v_{ci}}{v_r - v_{ci}}, & v_{ci} \le v_{aw} \le v_r \\ P_{rated}, & v_r \le v_{aw} \le v_{co} \\ 0, & v_{co} \le v_{aw} \end{cases}$$
(22)

Where  $P_{rated}$  is wind turbine rated power,  $v_{ci}$ ,  $v_r$ ,  $v_{co}$  are respectively cut in speed, rated speed, and cut-off speed of the wind turbine and  $v_{aw}$  is average wind speed of state.

Eventually, wind turbine average power is calculated using Eq. (23).

$$P_{av} = \sum_{v_{ci}}^{v_{co}} P_{out}(v) \times P_{v} \{G_{w}\}$$
(23)

### 5.2. Uncertainty Modeling of Solar Panel Generated Power

The output power of solar panels depends largely on the amount of solar radiation level. Hourly distribution of solar radiation at a given time follows the beta probability distribution function that it is obtained as Eq. (24) [18].

$$f_{b}(s) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} \times s^{\alpha - 1} \times (1 - s)^{(\beta - 1)}$$

$$0 \le s \le 1, \alpha \ge 0, \beta \ge 0$$
(24)

Where s is solar irradiance  $(kW/m^2)$ ,  $\beta$ , $\alpha$  are parameters of the Beta distribution function that obtained as follows [18]:

$$\beta = (1 - \mu) \times \left(\frac{\mu \times (1 + \mu)}{\sigma^2} - 1\right)$$
(25)

$$\alpha = \frac{\mu \times \beta}{1 - \mu} \tag{26}$$

That  $\mu$  and  $\sigma$  are the mean and standard deviation of solar irradiance respectively. Due to continuous changes in solar irradiance, its changes is divided in to interval of 0.1 (kW/m<sup>2</sup>) and the middle of each interval is considered as its representative. So the probability of solar irradiance for each part is obtained according to Eq. (27) [18].

$$P_{s}\left\{G_{y}\right\} = \int_{s_{y1}}^{s_{y2}} f_{b}(s)ds$$

$$\tag{27}$$

Where  $f_b(s)$  is solar irradiance probability distribution function,  $s_{y1}, s_{y2}$  are solar irradiance limits of state y. There is a linear relationship between solar irradiance and out power of solar panel that obtained as following equations [18].

$$T_{cy} = T_A + s_{ay} \left[ \frac{N_{oT} - 20}{0.8} \right]$$
 (28)

$$I_{y} = s_{ay} [I_{sc} + K_{i} (T_{c} - 25)]$$
(29)

$$V_{y} = V_{oc} - K_{y} \times T_{cy}$$
(30)

$$FF = \frac{V_{MPP} \times I_{MPP}}{V_{oc} \times I_{sc}}$$
(31)

$$P_{sy}(s_{ay}) = N \times FF \times V_{y} \times I_{y}$$
(32)

Where

- $T_{cv}$  Cell temperature during state y (°C);
- $T_A$  Environment temperature (°C);
- $K_v$  Voltage temperature coefficient (V/°C);

#### Current temperature coefficient $(A/^{\circ}C)$ ; K.

- Nor Cell nominal temperature ( $^{\circ}$ C);
- FF Fill factor:
- Short circuit current (A); I
- V Open circuit voltage (V);
- Voltage at the maximum power point (V); V<sub>MPP</sub>
- Current at the maximum power point (A); I<sub>MPP</sub>
- Average solar irradiance of state  $y(kw/m^2)$ ; S<sub>av</sub>
- Output power of the module during state y(W). P<sub>sv</sub>

So, the average power of solar panel is obtained as follows:

$$P_{av} = \sum_{s_{y1}}^{s_{y2}} P_{sy}(s_{ay}) \times P_s \{G_y\}$$
(33)

#### 6. **Modeling Costs Related to Distributed Generations**

Distribution companies are obligated to deliver power demand with the best quality and lowest cost to consumers. Thus, in this part different costs of DG sources are modeled [19].

$$C_1 = \sum_{i=1}^{N_{DG}} Cost_{inst, DGi}$$
(34)

$$C_2 = \sum_{i=1}^{N_{DG}} Cost_{main, DGi}$$
(35)

$$CPV(C_2) = C_2 \times \sum_{r=1}^{T} \left( \frac{1 + InfR}{1 + IntR} \right)^r$$
(36)

$$C_{3} = \sum_{i=1}^{N_{DG}} \sum_{j=1}^{n} T_{j} \times DG_{j,i} \times CG_{i}$$
(37)

$$CPV(C_3) = C_3 \times \sum_{t=1}^{T} \left(\frac{1 + InfR}{1 + IntR}\right)^t$$
(38)

Where

| <b>C</b> <sub>1</sub> | Investment cost;                           |
|-----------------------|--|
| $N_{\text{DG}}$       | Number of DG unit;                         |
| Cost <sub>inst</sub>  | Investment cost of DG sources (\$/MW);     |
| C <sub>2</sub>        | Maintenance cost;                          |
| Cost_main             | Maintenance cost of DG sources (\$/MWh);   |
| InfR                  | Inflation rate;                            |
| IntR                  | Interest rate;                             |
| C <sub>3</sub>        | Operation cost;                            |
| $DG_{j,i}$            | Generated power by DG source in identified |
|                       | load level (MW);                           |
| CG <sub>i</sub>       | Operation cost of DG sources (\$/MWh);     |
| $\mathbf{T}_{j}$      | Passing time (h/year);                     |
| CDV                   | Continue and a still                       |

CPV() Cost present worth.

#### Modeling Benefit From The Presence of Distributed 7. Generations

Part of the power supply by the distribution companies to consumers is wasted in the transmission line and the rest of its reaches to consumers. So in this section, benefits from power reduction which purchased from the network and network lines loss reduction due to the presence of DGs has been explained [19].

$$\Delta Loss_{j,i} = \sum_{i=1}^{NDG} \left( Loss_{NDG,j} - Loss_{DG_{j,i}} \right)$$
(39)

$$\Delta PT = \Delta Loss_{j,i} + \sum_{i=1}^{NDG} DG_{j,i}$$

$$\tag{40}$$

$$B_1 = \sum_{j=1}^{n} C_{MWh,j} \times \Delta PT \times T_j$$
(41)

$$BPV(B_{1}) = B_{1} \times \sum_{n=1}^{T} (\frac{1 + InfR}{1 + IntR})^{n}$$
(42)

Where

| $\Delta Loss_{j,i}$      | Network loss reduction (MW);  |
|--------------------------|---|
| $N_{\text{DG}}$          | Number of DG unit;  |
| $Loss_{_{NDG,j}}$        | Network loss without DG (MW);                                       |
| $\text{Loss}_{DG_{j,i}}$ | Network loss with DG (MW);  |
| ΔΡΤ                      | The reduction in active power purchased from the transmission line; |
| $DG_{j,i}$               | Generated power by DG source in identified                          |
|                          | load level (MW);  |
| $\mathbf{B}_{1}$         | Active power reduction benefit                                      |
|                          | for each year;  |
| $C_{_{MWh,j}}$           | Energy market price in load level j (\$/MWh);                       |
| $T_{j}$                  | Passing time (h/year);  |
| $BPV(B_1)$               | Benefit present worth;  |
| InfR                     | Inflation rate;   |
| IntR                     | Interest rate.  |

As a result, based on the costs and benefits described, the net profit is defined as follows:

$$Saving = BPV(B_1) - [C_1 + CPV(C_2) + CPV(C_3)]$$
(43)

#### Improved Particle Swarm Optimization Algorithm 8.

IPSO§ algorithm is improved PSO\*\* algorithm, so in this section, first PSO is briefly described.

PSO is an optimization method that was first developed by Eberhart and Kennedy [20]. This algorithm has been inspired from the mass movement of birds that they are looking for food and each bird is called a particle in the search space. Each particle has two values, which update at each stage of the movement. These values are position  $(X_i)$  and speed  $(V_i)$  that update with the better of  $P_{best}$  and

<sup>§</sup> Improved Particle Swarm Optimization

<sup>\*\*</sup> Particle Swarm Optimization

 $G_{best}$  which respectively named, the best value so far obtained for each particle and the best value ever achieved by all particles among the total population. In each iteration after finding  $P_{best}$  and  $G_{best}$ , speed and place of each particle update from Eq. (44) and Eq. (45).

$$V_{i}^{k+1} = wV_{i}^{k} + c_{1}r_{1} + (P_{besti}^{k} - X_{i}^{k}) + c_{2}r_{2} + (G_{besti}^{k} - X_{i}^{k})$$
(44)  
$$X_{i}^{k+1} = X_{i}^{k} + V_{i}^{k+1}$$
(45)

Where

 $V_i^k$  Speed of the particle i in iteration-k;

 $c_1, c_2$  Acceleration coefficient in the range of [1, 2];

- $\mathbf{r}_{1},\mathbf{r}_{2}$  Random numbers in the range of [0, 1];
- $X_i^k$  Position of the particle in iteration-k;
- $P_{\text{best}i}^{k}$  The best position of the particle in iteration-k;

 $G_{\text{best}i}^{k}$  The best position of the group in iteration-k.

And w is the inertia weight that defined as follows:

$$w = w_{\max} - \frac{w_{\max} - w_{\min}}{Iter_{\max}} \times Iter$$
(46)

Where

w<sub>min</sub> Lowest weight;

w<sub>max</sub> Highest weight;

Iter<sub>max</sub> The maximum number of iteration;

Iter The iteration number.

Often the dynamic range of variables in PSO algorithm are set to  $V_{max}$ . According to the above-mentioned subjects, inertia weight parameter (w) controls speed (V). If  $V_{max}$  is too large, particles away from previous good response and if  $V_{max}$  is too small particles do not discover a better local responses.

So IPSO algorithm define variable weight parameter which can be passed from local minimum that obtained as follows [21]:

 $W_{new} = W \cdot D \tag{47}$ 

D is an irregular parameter that defined as follows in iteration-k.

 $D_{k} = \mu \cdot D_{k-1} \cdot (1 - D_{k-1}) \tag{48}$ 

Which  $\mu$  is a control parameter that its amount is changed in the range of [0, 4]. When  $\mu = 4$  and  $D_0 = \{0, 0.25, 0.50, 0.75, 1\}$  the relation shows irregular behavior.

## 9. Improved Particle Swarm Optimization Algorithm Implementation

In this section to achieve the mentioned purposes, optimal location of DGs obtained using IPSO algorithm.

- 1. The amount of DGs' generated power that is obtained due to the methods of uncertainty, is considered as input parameters of the algorithm.
- 2. The first generation of particles is produced in random order (first place candidates) and  $P_{best}$  and  $G_{best}$  are recorded.
- 3. Linear three-phase unbalanced load flow run and network constraints are examined.
- The next generation is produced, if the number of iterations reach to the specified value and network constraints are not violated, G<sub>best</sub> selected and location of DG is introduced and the objective function is calculated. Otherwise, the algorithm is repeated so as to reach the maximum number of iterations.

## 10. Case Study

The proposed algorithm is supposed to be applied on 4.8 (kV), 37-bus radial unbalanced distribution network that its load connection type is delta (D) and load models are constant power (PQ), constant current (I) and constant impedance (Z) that shown in Fig. 1 [22]. Simulations have been performed regardless of network transformator and regulator. So buses with 799 and 775 index are neglected and the number of buses reduced to 35.

## 10.1. The Information Required For Simulation

The proposed work has been implemented in MATLAB R2010a computing environment with Core i5, 2.20 GHz computer with 8.00 GB RAM.

Coefficients of multi-objective function are selected as  $k_1 = k_2 = 0.3$  and  $k_3 = 0.4$  that gives the best results. The maximum number of iterations is considered 100.

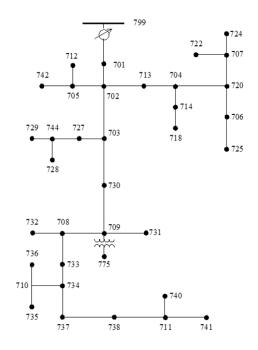


Fig. 1. IEEE 37 bus unbalanced network [22].

DGs Commercial information and the price of power purchased from the transmission line at a basic load level are respectively shown in Table 1 and Table 2. It is noteworthy that DGs are considered as the negative node PQ (negatively load). The parameters for modeling generated power uncertainty of wind turbines and solar panels are respectively shown in Table 3 and Table 4. In wind turbine k=3 and c=8 and in solar panel  $\alpha,\beta=3.5$  have been considered.

 Table 1. DGs Commercial information [19]

| Parameter            | Unit   | Value  |
|----------------------|--------|--------|
| DG installation cost | \$/MW  | 318000 |
| DG operation cost    | \$/MWh | 29     |
| DG maintenance cost  | \$/MWh | 7      |
| Interest rate        | %      | 12.5   |
| Inflation rate       | %      | 9      |
| DG Life time         | Year   | 20     |

 Table 2. Technical and Commercial Information of Loads
 [19]

| Network situation       | Peak load |
|-------------------------|-----------|
| Percentage of peak load | 95-100    |
| Time duration (h/year)  | 1825      |
| Market price (\$/MWh)   | 70        |

Table 3. Characteristics of the wind turbines [23]

| Wind turbine characteristics | Turbine 1 | Turbine 2 |
|------------------------------|-----------|-----------|
| Rated power                  | 300 KW    | 600 KW    |
| Cut-in speed (m/s)           | 2.5       | 4         |
| Rated speed (m/s)            | 13        | 16        |
| Cut-out speed (m/s)          | 25        | 20        |

Table 4. Characteristics of the PV modules [23]

| PV module characteristics                        | PV1   | PV2   |
|--|-------|-------|
| Open circuit voltage (V)                         | 21.70 | 36.96 |
| Short circuit current (A)                        | 3.40  | 8.38  |
| Voltage at maximum power (V)                     | 17.40 | 28.36 |
| Current at maximum power (A)                     | 3.05  | 7.76  |
| Voltage temperature coefficient $(mV/^{\circ}C)$ | 88    | 127.8 |
| Current temperature coefficient (mA/° C)         | 1.05  | 5.45  |
| Nominal cell operating temperature<br>(°C)       | 43    | 43    |

## 10.2. Simulation Results

The network is unbalanced due to unequal loads of phases and asymmetrical lines. So single phase DGs must be installed in one of the phases to achieve optimum results. According to Table 3 and 4, Table 5 shows the average power of DGs.

| Type of distributed generations | P <sub>ave</sub> (kw) |
|---------------------------------|-----------------------|
| Turbine 1                       | 126.1                 |
| Turbine 2                       | 152.2                 |
| PV1                             | 25.91                 |
| PV2                             | 60.32                 |

By using IPSO algorithm and linear three-phase unbalanced load flow method, suitable buses and phases to install wind turbines and solar panels that their capacity was mentioned in Table 5, can be obtained.

It is noteworthy that in this paper, loads are considered at the level of peak load and variables. Loads model and connection type are considered based on the IEEE standard.

Suitable phases and buses to install DGs are shown in Table 6 and the obtained results for network power losses and maximum percentage of voltage unbalance factor, with and without DG installation are shown in Table 7.

Table 6. Suitable phases and buses for DG installation

| Bus Number | Bus Phase | Type of DG |
|------------|-----------|------------|
| 714        | В         | Turbine 1  |
| 727        | С         | PV1        |
| 733        | А         | PV2        |
| 740        | С         | Turbine 2  |

Table 7. Comparison results with and without DGs

| P <sub>Loss</sub> (1 | KW)      | VUF <sub>max</sub> (%) |        |  |  |  |
|----------------------|----------|------------------------|--------|--|--|--|
| Without DG           | With DGs | Without DG With DGs    |        |  |  |  |
| 33.1192              | 22.2557  | 1.3816                 | 1.3065 |  |  |  |

According to the obtained results from Table 7, network active power loss and maximum percentage of voltage unbalance factor are reduced after installing four DGs.

After determination the appropriate location and size of DGs, different costs and also benefit and saving based on Table 1 and Table 2 obtained that shows in Table 8.

 Table 8. The results of economic productivity of DGs allocation

| Network condition (\$) | Value    |
|------------------------|----------|
| Investment cost        | 1272000  |
| Maintenance cost       | 408.5574 |
| Operation cost         | 2718000  |
| Benefit                | 9572600  |
| Saving                 | 5582200  |

The cost of the DG units itself, the cost of research, preparation of places for installing DGs and construction are part of the distributed generation units investment costs. Maintenance cost include electrical and mechanical check and repair.

Input costs such as fuel costs of these source for power generation are part of the operational costs.

After obtaining various costs and benefit, saving is obtained from the difference between benefit and costs.

It should be noted that what time after installing DGs, profitability achieved. Thus, the amount of profit after installing four DGs is given in Fig. 2, for per year in the period of 20 years.

The time required to start optimal placement profitability is obtained from cessation of the curve with the zero axis.

The studied network is unbalanced so phases voltage size and angle can not be equal. In Table 9 phases voltage is shown before and after the installation of DGs and in Table 10 comparison between the phases voltage angle before and after the installation of DGs has been done.

Percentage of voltage unbalance factor of each bus with and without DG placement has been compared in Fig. 3.

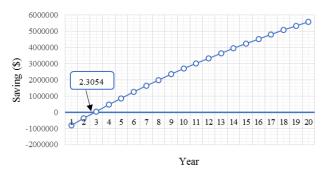


Fig. 2. Profitability process with 4 DG for 20 years.

As shown in Fig. 2, can be seen that profitability will be started after 2 years, 3 months and 21 days.

| Table 9.  | Com | parison | between | size | (perunit) | ) of | different | phase | voltages | with ar | d withou  | t DGs |
|-----------|-----|---------|---------|------|-----------|------|-----------|-------|----------|---------|-----------|-------|
| I abic 7. | Com | parison | between | SILC | (perunit  | , 01 | uniterent | phase | vonages  | with a  | ia withou | 1 003 |

| Bus | Bus    |                    | Without DG                |                    |         | With 4 DG                                      |                             |
|-----|--------|--------------------|---------------------------|--------------------|---------|--|-----------------------------|
| no. | Number | $ \mathbf{V}_{A} $ | $ \mathbf{V}_{\text{B}} $ | $ \mathbf{V}_{c} $ | $ V_A $ | $ \mathbf{V}_{\scriptscriptstyle \mathrm{B}} $ | $ \mathbf{V}_{\mathrm{c}} $ |
| 1   | 701    | 1.0317             | 1.0144                    | 1.0183             | 1.0317  | 1.0144   | 1.0183                      |
| 2   | 702    | 1.0247             | 1.0088                    | 1.0101             | 1.0256  | 1.0094   | 1.0115                      |
| 3   | 703    | 1.0178             | 1.005                     | 1.0034             | 1.02    | 1.0056   | 1.0062                      |
| 4   | 704    | 1.0217             | 1.0044                    | 1.0064             | 1.0226  | 1.0064   | 1.0087                      |
| 5   | 705    | 1.0241             | 1.0075                    | 1.0088             | 1.025   | 1.008  | 1.0102                      |
| 6   | 706    | 1.0204             | 1.0006                    | 1.0039             | 1.0213  | 1.0027   | 1.0061                      |
| 7   | 707    | 1.0187             | 0.9959                    | 1.0024             | 1.0196  | 0.9979   | 1.0047                      |
| 8   | 708    | 1.0086             | 1.0002                    | 0.9945             | 1.013   | 1.0011   | 0.9996                      |
| 9   | 709    | 1.0111             | 1.0012                    | 0.9967             | 1.0148  | 1.002  | 1.0011                      |
| 10  | 710    | 1.0024             | 0.9967                    | 0.9879             | 1.0079  | 0.9977   | 0.9948                      |
| 11  | 711    | 0.9982             | 0.9962                    | 0.9853             | 1.0053  | 0.9972   | 0.9953                      |
| 12  | 712    | 1.024              | 1.0073                    | 1.0082             | 1.0248  | 1.0079   | 1.0096                      |
| 13  | 713    | 1.0234             | 1.0069                    | 1.0083             | 1.0243  | 1.0081   | 1.01                        |
| 14  | 714    | 1.0214             | 1.0043                    | 1.0064             | 1.0224  | 1.0066   | 1.0087                      |
| 15  | 718    | 1.0201             | 1.0041                    | 1.0059             | 1.021   | 1.0064   | 1.0083                      |
| 16  | 720    | 1.0205             | 1.0011                    | 1.004              | 1.0214  | 1.0031   | 1.0063                      |
| 17  | 722    | 1.0185             | 0.9953                    | 1.0023             | 1.0194  | 0.9974   | 1.0045                      |
| 18  | 724    | 1.0184             | 0.9949                    | 1.0023             | 1.0193  | 0.997  | 1.0045                      |
| 19  | 725    | 1.0202             | 1.0003                    | 1.0038             | 1.0212  | 1.0023   | 1.0061                      |
| 20  | 727    | 1.0167             | 1.0044                    | 1.0025             | 1.019   | 1.0049   | 1.0054                      |
| 21  | 728    | 1.0156             | 1.0036                    | 1.0017             | 1.0179  | 1.0042   | 1.0046                      |
| 22  | 729    | 1.0156             | 1.0039                    | 1.002              | 1.0179  | 1.0045   | 1.0049                      |
| 23  | 730    | 1.0127             | 1.0021                    | 0.9982             | 1.0161  | 1.0028   | 1.0022                      |
| 24  | 731    | 1.0109             | 1.0003                    | 0.9965             | 1.0146  | 1.0011   | 1.0009                      |
| 25  | 732    | 1.0086             | 1.0001                    | 0.9941             | 1.0129  | 1.001  | 0.9992                      |
| 26  | 733    | 1.0063             | 0.9992                    | 0.9926             | 1.0112  | 1.0002   | 0.9984                      |
| 27  | 734    | 1.0029             | 0.9977                    | 0.9894             | 1.0084  | 0.9987   | 0.9963                      |
| 28  | 735    | 1.0023             | 0.9965                    | 0.9873             | 1.0078  | 0.9975   | 0.9943                      |
| 29  | 736    | 1.0018             | 0.9951                    | 0.9875             | 1.0074  | 0.9961   | 0.9945                      |
| 30  | 737    | 0.9996             | 0.9968                    | 0.9873             | 1.0058  | 0.9978   | 0.9956                      |
| 31  | 738    | 0.9984             | 0.9964                    | 0.9862             | 1.0052  | 0.9974   | 0.9953                      |
| 32  | 740    | 0.9981             | 0.9961                    | 0.9847             | 1.0055  | 0.9971   | 0.9955                      |
| 33  | 741    | 0.9981             | 0.9962                    | 0.985              | 1.0052  | 0.9971   | 0.995                       |
| 34  | 742    | 1.0238             | 1.0066                    | 1.0086             | 1.0246  | 1.0072   | 1.0101                      |
| 35  | 744    | 1.016              | 1.004                     | 1.0021             | 1.0183  | 1.0046   | 1.005                       |

| Bus | Bus    |              | Without DG                      |                  |              | With 4 DG                       |                  |
|-----|--------|--------------|---------------------------------|------------------|--------------|---------------------------------|------------------|
| no. | Number | $\theta_{A}$ | $\theta_{\scriptscriptstyle B}$ | $\theta_{\rm c}$ | $\theta_{A}$ | $\theta_{\scriptscriptstyle B}$ | $\theta_{\rm c}$ |
| 1   | 701    | -0.0795      | -120.3894                       | 120.6089         | -0.0795      | -120.3894                       | 120.6089         |
| 2   | 702    | -0.1414      | -120.5827                       | 120.4231         | -0.1155      | -120.5098                       | 120.4855         |
| 3   | 703    | -0.1801      | -120.704                        | 120.1924         | -0.1447      | -120.5801                       | 120.3518         |
| 4   | 704    | -0.1796      | -120.6157                       | 120.4562         | -0.1098      | -120.4953                       | 120.4945         |
| 5   | 705    | -0.1352      | -120.5961                       | 120.4543         | -0.1094      | -120.5232                       | 120.5166         |
| 6   | 706    | -0.2239      | -120.6649                       | 120.5324         | -0.154       | -120.544                        | 120.5705         |
| 7   | 707    | -0.3034      | -120.6343                       | 120.6629         | -0.2334      | -120.513                        | 120.7009         |
| 8   | 708    | -0.0857      | -120.7371                       | 120.0126         | -0.06        | -120.5492                       | 120.2876         |
| 9   | 709    | -0.1144      | -120.7356                       | 120.0706         | -0.0859      | -120.566                        | 120.3123         |
| 10  | 710    | 0.0039       | -120.7684                       | 119.9043         | 0.007        | -120.5218                       | 120.2556         |
| 11  | 711    | 0.049        | -120.742                        | 119.7509         | 0.004        | -120.393                        | 120.213          |
| 12  | 712    | -0.1193      | -120.6107                       | 120.4579         | -0.0935      | -120.5377                       | 120.5202         |
| 13  | 713    | -0.1499      | -120.6046                       | 120.4345         | -0.106       | -120.5123                       | 120.487          |
| 14  | 714    | -0.1787      | -120.6089                       | 120.4517         | -0.1016      | -120.4836                       | 120.4848         |
| 15  | 718    | -0.168       | -120.5735                       | 120.4198         | -0.0909      | -120.4483                       | 120.4529         |
| 16  | 720    | -0.2143      | -120.664                        | 120.5216         | -0.1445      | -120.5431                       | 120.5598         |
| 17  | 722    | -0.3119      | -120.6318                       | 120.6773         | -0.2419      | -120.5105                       | 120.7152         |
| 18  | 724    | -0.323       | -120.6252                       | 120.6892         | -0.2531      | -120.5039                       | 120.7272         |
| 19  | 725    | -0.2311      | -120.6615                       | 120.5421         | -0.1612      | -120.5406                       | 120.5802         |
| 20  | 727    | -0.1618      | -120.6916                       | 120.184          | -0.1296      | -120.5629                       | 120.3467         |
| 21  | 728    | -0.1586      | -120.6807                       | 120.1744         | -0.1265      | -120.552                        | 120.3372         |
| 22  | 729    | -0.1595      | -120.6743                       | 120.163          | -0.1274      | -120.5456                       | 120.3259         |
| 23  | 730    | -0.1258      | -120.7337                       | 120.0997         | -0.0955      | -120.5756                       | 120.3207         |
| 24  | 731    | -0.1339      | -120.7385                       | 120.0918         | -0.1054      | -120.5689                       | 120.3335         |
| 25  | 732    | -0.0746      | -120.7469                       | 120.0156         | -0.0491      | -120.5589                       | 120.2906         |
| 26  | 733    | -0.0624      | -120.7322                       | 119.9556         | -0.0396      | -120.5259                       | 120.2641         |
| 27  | 734    | -0.0185      | -120.7421                       | 119.8782         | -0.0148      | -120.4957                       | 120.2297         |
| 28  | 735    | 0.0178       | -120.7809                       | 119.9074         | 0.0207       | -120.5342                       | 120.2587         |
| 29  | 736    | -0.0295      | -120.7531                       | 119.949          | -0.0264      | -120.5065                       | 120.3001         |
| 30  | 737    | 0.0085       | -120.7133                       | 119.7872         | -0.0094      | -120.4214                       | 120.1882         |
| 31  | 738    | 0.0277       | -120.7176                       | 119.757          | -0.0037      | -120.3972                       | 120.1887         |
| 32  | 740    | 0.0629       | -120.7545                       | 119.7541         | 0.0019       | -120.3813                       | 120.2323         |
| 33  | 741    | 0.056        | -120.7501                       | 119.7491         | 0.011        | -120.401                        | 120.2112         |
| 34  | 742    | -0.1513      | -120.5874                       | 120.4745         | -0.1255      | -120.5144                       | 120.5368         |
| 35  | 744    | -0.1626      | -120.6837                       | 120.1714         | -0.1305      | -120.555                        | 120.3342         |

Table 10. Comparison between angle (degree) of different phase voltages with and without DGs

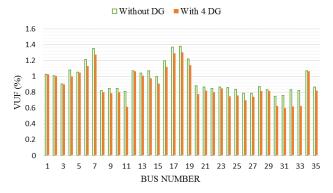


Fig. 3. Buses VUF before and after DGs allocation.

By comparing percentage of voltage unbalance factor before and after installing DGs as shown in Fig. 3 can be seen that, VUF have fallen after DG placement, which indicates that the results obtained properly. Table 9 shows that the voltage of each phase after installing DGs improved.

### 11. Conclusion

Unequal phase loads and asymmetric structure of lines are made distribution networks actually unbalanced. Therefore, efforts must be made to reduce the network unbalanced with appropriate methods. In this paper, the effects of distributed generations investigated on the 37 bus unbalanced network. Since the generated power of wind turbines and solar panels are dependent on external factors, initially, the generated power uncertainty of these sources has been modeled. Then the various costs and benefits of installing DGs has been expressed. Due to the need for accurate load flow in distribution networks, linear threephase unbalanced load flow is used where the loads model and connection type of loads are considered based on the

IEEE standard. Then IPSO algorithm that is one of the optimization methods has been described. So first size of the four wind and solar DGs obtained based on uncertainty methods then the exact location and phase of DGs selected by weighted multi-objective IPSO algorithm. The simulations have been done in Matlab software. DGs placement reduce power loss and percentage of voltage unbalance factor, improve voltage profile, and enhance the profitability of distribution companies.

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