A Hybrid Game Theoretic approach to Generator Bidding in Energy and Ancillary Service Markets

Durga Hari Kiran B *‡, Sailaja Kumari M*

*Department of Electrical Engineering, National Institute of Technology Warangal, Warangal, India.

(bdurgaharikiran@yahoo.co.uk, sailaja_matam@yahoo.com)

‡ Corresponding Author; Durga Hari Kiran B, Research Scholar, Department of Electrical Engineering, National Institute of Technology Warangal, Warangal, India. Tel: +91 9989772198, bdurgaharikiran@yahoo.co.uk

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Abstract- This paper proposes a solution to generator bidding strategy using a novel hybrid Evolutionary Game Theory (EGT) and Differential Evolution (DE) method. In restructured power system, the generating companies (GENCOs) have an opportunity to compete in energy and ancillary services markets and earn profits. This competition creates a complicated situation to System Operator (SO) in the market clearing process. This paper attempts to maximize GENCOs profit with incomplete information by adopting optimal bidding strategies in energy and ancillary service markets while considering unit commitment constraints. Supply Function Equilibrium (SFE) model is employed to compute GENCOs profit. Nash Equilibrium points were calculated in the first stage by using Evolutionary Game Theory and then optimal bidding strategies were found with the help of Differential Evolution method. Evolutionary Game Theory is best suited for GENCOs bidding strategies but leads to slow convergence due to a large number of variables. So, a novel hybrid method involving Evolutionary Game Theory with Differential Evolution is proposed in this paper. The proposed method to solve bidding strategies is employed on WSCC 9 and New England 39 bus test systems to demonstrate its merits.

Keywords Bidding, Non-cooperative, Game theory, evolutionary programming, unit commitment, supply function equilibrium, Nash Equilibrium.

1. Introduction

Under the restructuring of power systems, GENCOs try to maximize their profit by bidding their generation capacity in power markets. Due to uncertainties in load, renewable energy integration and incomplete information of other participants, GENCOs often end up producing at less profit or half of their full capacity. This problem can be overcome by allowing GENCOs to participate in both Energy and Ancillary Services markets which give a window of opportunity for them to bid and provide ancillary services competitively. Market Clearing Price (MCP) will be computed from the bids placed by the GENCOs and system operator clears both the markets based on this price. If a GENCO tries to bid higher than MCP, then such bid will be eliminated for few hours.

The strategies of GENCOs to bid in energy market depends mainly on the other participating generator information, load uncertainties, and renewable, e.g., wind integration into the grid. For ancillary services market, GENCOs bidding depends on the amount of reserve and reactive power required. Many methods based on game and non-game are used to determine the strategies for generation companies to bid.

The classification for game-based methods is given in [1], in which a non-cooperative game theory based method is developed, and strategic bidding among transmission constrained generators is attempted. In [2] to obtain supply function equilibrium for producers, Improved Prey—Predator Optimization algorithm was proposed to find the optimal strategies for GENCOs bidding. Additionally, a scenario based market clearing method was proposed. In [3] bat-inspired algorithm solution to optimal bidding strategies based on linear supply function equilibrium of GENCOs was proposed for network-constrained electricity markets. An optimal stochastic optimization problem of bidding and scheduling of batteries was proposed in [4].

[5] has proposed a price taker bidding for power generators in the Turkish power market. In [6] authors have proposed strategies to sell power and reserve in deregulated

market with Price Based Unit Commitment Constraints (PBUC). They proposed a Binary Fish Swarm Algorithm to solve PBUC problem. A mathematical model to solve strategic bidding for the large consumer under smart grid environment was proposed in [6] to achieve minimum cost for procuring energy. In [7] Bayesian Nash Equilibrium points are found simultaneously to determine optimum bidding parameters for GENCOs in Energy and Reactive power markets. [8] proposed a Cournot model to solve generator bidding strategies in a day-ahead oligopoly energy markets under bilateral contracts by considering demand elasticity, network security constraints. [1,7] solved the bidding problem for generators with non-cooperative game theory method. [2-6] proposed heuristic methods for resolve the bidding problem. The unit commitment constraints are not considered in some of the studies, some have assumed the incomplete information of participating GENCOs from the historical data, and some have done the bidding for energy market alone. Bidding in ancillary services markets brings significant profits for generators. [9] has presented optimal bidding strategies for generating companies based on parametric linear programming using incomplete information. [10] highlighted some insights in ancillary services bidding in ERCOT market.

Future markets are going to incorporate renewable into the grid makes this study on generation bidding interesting, as the grid requires more and more operating reserve support. In [11] authors investigated the importance of Ancillary Service market in wind integrated markets. In [12] Rasool Kazemzadeh et al. assessed the impact of wind uncertainty on energy and ancillary services markets using evaluation matrices. In [13] S. Souag et al. proposed Economic Dispatch Enhancement by considering ramping constraint in wind integarted Algerian power grid.

In this paper to determine optimal bidding strategies in energy and ancillary services markets, a hybrid method consisting of non-cooperative Evolutionary Game Theory and Differential Evolution method is used. It is considered that generators compete in 24-hour day-ahead market limited by unit commitment constraints. In this paper to get profitable and robust bidding strategies, the generators are considered to know only generator coefficients. Game theory or evolutionary computing methods alone cannot solve this complex optimization problem when generators with incomplete information and generator constraints are considered. So, a new hybrid method is proposed in this paper for profitable bidding of GENCOs by considering unit commitment constraints.

In power market structure, participants (GENCOs) strategic bidding may follow Bertrand, Cournot, Stackelberg, and Supply Function Equilibrium (SFE) economic models. [1] explained the significance of the above four models and why SFE model is most suitable for generator bidding. Generators bidding strategies must closely resemble the actual player actions in the power market. To associate the bidding quantity with the bid price, SFE model with incomplete information is employed in this paper.

The major contributions of this paper are 1) 24-hour schedule for generators and bidding strategies are obtained with incomplete information and unit commitment constraints. 2) A new solution method involving hybrid non-cooperative Evolutionary Game Theory and Differential Evolution is proposed to get the optimal bidding strategies of Generators in energy and ancillary service markets.

The paper is organized as follows: Section 1 presents introduction and literature survey. Section 2 deals with the problem formulation. Section 3 gives the solution to unit commitment problem considered using Lagrangian Relaxation method Section 4 describes hybrid Evolutionary Game Theory method and Differential Evolution Method. Section 5 presents outcomes of the work and discussions. Finally, Section 6 gives the contribution of the work and Section 7 provides contributions of the work with concluding remarks.

2. Problem Formulation

The markets in restructured power system carry out coordinated activities. They need to assess the Energy and Ancillary Service markets simultaneously. Optimal bidding strategies help markets to balance the supply and demand better. Energy markets are classified into day-ahead, hour-ahead and real-time markets based on their time of operation in the power system. In these markets, System Operator (SO) determines price points for every instant with the help of complete system model. These price points give an opportunity to SO to assess the reliability needs and to mitigate infeasible bids. In this paper, the solution to optimal bidding strategies is solved using Hybrid Evolutionary Game Theory technique by imposing unit commitment constraints and feasible unit schedules are found in a Day-a-head market. Unit commitment allows SO to procure least cost resources to meet uncertainties in load, renewable energy integration and incomplete information of other participants. The problem model is discussed in the following sections, to find the optimal solution.

2.1 Modeling GENCOs incomplete information

To obtain the best bidding parameters for profitable bidding GENCOs needed to know the other GENCOs information. But mostly they do not have full access to opponent's characteristics. So, in this paper incomplete information of GENCOs is designed as follows:

1) All players (GENCOs) considered having only thermal units.
2) Fuel type and minimum-maximum generation levels of their opponents are known and

The solution to incomplete information games comes under Bayesian Nash Equilibrium [1]. For a generator i fuel cost (F.C) is expressed as a quadratic function of its active power generation in \$/hr, given in the Eq. (1) where a,b,c are.

$$F.C(Pg_{i,k}) = c + b * Pg_{i,k} + a * Pg_{i,k}^2 \quad (1)$$

where Pg_i is the active power generated by generator i in k^{th} GENCO. The Marginal Cost (M.C) of a generator i, which is the first order derivate of Eq. (1), is given in Eq. (2) in \$/MWhr

$$M.C = \lambda_{i,k} = b + 2 * a * Pg_{i,k} \quad (2)$$

2.2 Bids submitted by GENCO in Energy and Ancillary Service Market

GENCOs can submit their quantity price curve as a piecewise curve as shown in the fig. (1) and single block price for Ancillary Service Markets as shown in the fig. (2). So, for the i^{th} generator, a kth GENCOs can create their bidding segments to participate in both the markets simultaneously.

$$B_{i,k,b} = S_{i,k,b} * (M.C_{i,k,b}) \quad (3)$$

where $B_{i,k,b}$ is the bid price for generator i and GENCO k in the Energy market for a bid block b, $S_{i,k,b}$ is the bidding strategy, $M.C_{i,k,b}$ is the market clearing price of generator i at bid block b for k^{th} GENCO.

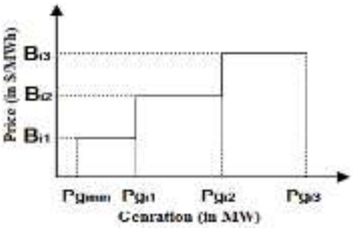

Fig. 1. Generators bidding curve in energy market

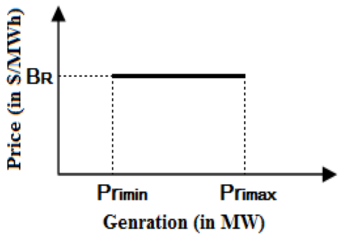

Fig. 2. Reserve bid block curve in Ancillary Service Markets

As the number of bidding blocks increases, the complexity of bidding increases. GENCOs can have any number of bidding blocks in Energy and Ancillary Service Market. But, to simulate the bidding problem, minimum numbers of bid blocks are chosen for Energy and Ancillary service markets. This paper considers that each GENCO competes with three bid blocks in Energy market shown in Fig. 1 and one bid block is shown in Fig. 2 for providing Operating Reserve in Ancillary Service market which was given by (4).

$$(Pg_{i,1} - Pg_{i,min}, B_{i,1}),(Pg_{i,2} - Pg_{i,1}, B_{i,2}),(Pg_{i,max} - Pg_{i,2}, B_{i,3}), (Pr_{i,min} - Pr_{i,max}, B_R) \quad (4)$$

Where Pg_{ix} is the power generated by the ith generator in block period x, Pr_{imin} and Pr_{imax} minimum and a maximum of limits for reserve provision by thermal generators, B_R is the bid price for generator i for providing Operating Reserve in Ancillary Service market.The generalized form of Eq. 4 for a kth GENCO can be written as Eq. 5.

$$(Pg_{i,k,1} - Pg_{i,k,min}, B_{i,k,1}),(Pg_{i,k,2} - Pg_{i,k,1}, B_{i,k,2}),(Pg_{i,k,max} - Pg_{i,k,2}, B_{i,k,3}), (Pr_{i,k,min} - Pr_{i,k,max}, B_{k,R}) \quad (5)$$

2.3 Unit commitment model

In Power markets, Energy and Operating Reserve Ancillary Service Markets operate simultaneously. GENCOs get maximum profit for the best bidding strategies being followed in these two markets. To make this happen GENCOs needed to model their opponent's strategies with the knowledge of power system conditions. In this paper, a two-level problem is solved. In the first stage, Nash Equilibrium points are identified and in the second stage SO solves for unit commitment problem. The SO will obtain the dispatched quantity of each GENCO concerning network conditions. The profit maximization objective is solved by Differential Evolution method, and the strategies of competing GENCOs where the information is incomplete are modeled using Evolutionary Game Theory method which is described in section 4. As the SO runs the unit commitment, the k^{th} term is absent in unit commitment problem considered in this section.

2.3.1 Objective function

The following co-optimization problem (Eq. 6) is solved for finding profit (PT) in both Energy and Ancillary Service Markets to get optimal bidding strategies of each GENCO.

$$Max(PT) = RE - TC \quad (6)$$

where RE is the revenue that the GENCO gets which is given as

$$RE = \max\left(\sum_{t=1}^{24} [\sum_{i=1}^{Ng} [\sum_{b=1}^{3} (M.C.P_{i,b,t} * Pg_{i,b,t}) + M.C.R_{i,t} * PR_{i,t}]] \right) \quad (7)$$

and TC is the total cost that incurs to a GENCO given as

$$TC = min \sum_{t=1}^{24} \sum_{i=1}^{Ng} [(1 - r) * F.C_i(Pg_{i,b,t}) + r * F.C_i(Pg_{i,b,t} + PR_{i,t})] * U_{i,t} + SU_{i,t} + SD_{i,t}] \quad (8)$$

where $M.C.P_{i,b,t}$ (in Eq. (7)) is the market clearing price paid to the generator i in bid block b for the hour t. $M.C.R_{i,t}$ market clearing price paid to generator i in hour t. N_g is the number of generators in the GENCO. The first term $F.C_i$ (.) (in Eq. (8)) is the cost paid to the thermal generators and the second term is a cost paid to the thermal generator for providing Reserve. The third and fourth term give start-up and shut-down cost. r is the probability of calling reserve. Considering the minimization of the cost paid to GENCO has a significant effect on Unit commitment schedule. The Unit commitment must satisfy several constraints to maintain power system under balance.

2.3.2 Problem constraints

Minimum-up time and down time constraints

The Minimum Up Time (M.U.T) and Minimum Down Time (M.D.T) limit of each unit 'i' and for a cycle 'c' are given as

$$\begin{aligned} &(X^{on}_{i,t-1} - \text{M.U.T}_i)(\text{U}_{i,t-1} - \text{U}_{i,t}) \geq 0 \\ &X^{on}_{i,t} = (X^{on}_{i,t-1} + 1)\text{U}_{i,t} \\ &(X^{off}_{i,t-1} - \text{M.D.T}_i)(\text{U}_{i,t} - \text{U}_{i,t-1}) \geq 0 \\ &X^{off}_{i,t} = (X^{off}_{i,t-1} + 1)(1 - \text{U}_{i,t}) \end{aligned} \tag{9}$$

Where X^{off}, X^{on} are the number of hour for which the unit is Off-line or On-line respectively.

Power Balance constraint

Eq. (10) gives the equality constraint to balance hourly power in the system.

$$\sum_{i=1}^{Ng} \sum_{t=1}^{24} [(Pg_{i,b,t} + PR_{i,t}) * U_{i,t}] = Pd_t \tag{10}$$

In Eq. (10) $Pg_{i,b,t}$ is power dispatched by committed thermal generator i in hour t. $U_{i,t}$ gives the status of the committed generator. $PR_{i,t}$ represents reserve requirement in hour t. Pd_t is the hourly demand. PR is provided by the same thermal unit which is committed.

Where $U_{i,t} = \begin{cases} 1 & ON \\ 0 & OFF \end{cases}$

The Operating Reserve Ancillary Service requirement constraint

Eq. (11) and (12) represent the operating reserve ancillary service constraint met by thermal generator i. Eq. (13) represents the reserve constraints.

$$\sum_{i=1}^{N_g} \sum_{t=1}^{24} [PR_{i,t} * U_{i,t}] \geq DPR_t \tag{11}$$

$$0 \leq PR_{i,t} \leq PR\,max_i \tag{12}$$

$$\begin{aligned} &PR_{i,t} \leq Ramp_i \\ &M.C.R_{i,t} \leq RR_{max} \end{aligned} \tag{13}$$

where DPR_t is the operating reserve requirement of the system in hour t, $PR\,max_i$ is the maximum operating reserve that a thermal unit can provide, $Ramp_i$ is the ramp rate of thermal generator i, RR_{max} is maximum price cap for reserve market.

Thermal Unit constraints

Eq. (14) and (15) represent thermal generator constraints for providing operating reserve ancillary service by thermal generator i in hour t alone.

$$Pg\,min_{i,t} \leq Pg_{i,t} \leq Pg\,max_{i,t} \tag{14}$$

$$Pg_{i,t} + PR_{i,t} \leq Pg\,max_{i,t} \tag{15}$$

where $Pg\,min_{i,t}$ and $Pg\,max_{i,t}$ are minimum and maximum limits of the thermal generator.

Other security constraints are also considered which are taken from [14]. From the optimal bidding strategies determined from Eq. (4) SO can determine Unit commitment and power dispatched by each committed generator.

After second stage GENCO has information of unit optimal schedule to participate in Energy and Ancillary service markets, thereby units update their bidding strategies until Nash Equilibrium is obtained. Solving this problem with Game theory alone or Evolutionary Computing methods is rigorous for the Operator. So, this paper has proposed a hybrid Evolutionary Game Theory method which is explained in section 4.

3. Lagrangian Relaxation

Unit commitment problem is solved by Lagrangian Relaxation (LR) method by "relaxing" the coupling constraints. The LR generates a separable problem by adding coupling constraints into the objective function; those coupling constraints are multiplied by "penalty factors" called as Lagrangian multipliers which will be determined iteratively. LR method is dependent on initial estimates of Lagrangian multipliers [15].

The objective function is to minimize total cost by considering optimal unit commitment schedule as given in Eq. (8). From the Eq. (8) the Lagrangian function of the single thermal unit is formed as

$$\begin{aligned} L(i) = &\sum_{i=1}^{N_g} \sum_{t=1}^{24} \begin{bmatrix} [(1-r) * F.C_i(Pg_{i,t}) + r * F.C_i(Pg_{i,t} + PR_{i,t})] * U_{i,t} \\ + SU_{i,t} + SD_{i,t} \end{bmatrix} \\ &- \sum_{i=1}^{N_g} \sum_{t=1}^{24} \lambda_d(t) * [(Pg_{i,t} + PR_{i,t}) * U_{i,t} - Pd_t] \\ &- \sum_{i=1}^{N_g} \sum_{t=1}^{24} \lambda_r(t) * [PR_{i,t} * U_{i,t} - DPR_t] \end{aligned} \tag{16}$$

Where λ_d, λ_r are the Lagrangian multipliers

The Lagrangian function is given in Eq. (16) for the thermal generator which is in ON state can be rewritten into Eq. (17) subjected to constraints given in Eq. (11)-(15) as

$$L(i)=\sum_{t=1}^{24}\left[F.C_i(Pg_{i,t})+r*PR_{i,t}+SU_{i,t}+SD_{i,t}\right] + \sum_{t=1}^{24}[Pd_t*(\lambda_d)+DPR_t*(\lambda_r)] - \sum_{t=1}^{24}Pg_{i,t}*(\lambda_d)-\sum_{t=1}^{T}PR_{i,t}*(\lambda_d+\lambda_r) \quad (17)$$

3.1 LR Algorithm

LR unit commitment approach is explained in the following algorithm.

1. Initialize Lagrange multipliers λ_d and λ_r
2. By using Dynamic Programming Solution to the Eq. 17 calculated, values for $Pg_{i,t}$ and $U_{i,t}$ are found.
3. Check for convergence if not met go to step 4.
4. Updated Lagrange multipliers are obtained using the gradient method.

4. Bidding Using Hybrid Evolutionary Game Theory Method

Research related to game theory application in power system problems are presented in [16-19]. In [16] imperfect competition among GENCOs is formulated using Nash-Cournot competition in Bilateral and poolco markets. But, the Nash-Cournot fails to model strategies among participants in a realistic way. [17] has presented multi-period Nash Equilibria in poolco based markets.[18] has found multiple Nash-Cournot Equilibria points in electricity market using a relaxation procedure. [19] has presented mixed integer linear problem for finding all Pure Nash Equilibria in a pool-based electricity market. The strategic bidding parameters for participating generating companies are obtained using game theory [1]. An optimal strategy is a Nash Equilibrium strategy for a GENCO when no other GENCO deviates from its best strategy [5, 6].

As discussed in section 2.1 and 2.2 to obtain optimal bidding parameters (i.e. B_{ij}, P_{i1-3}, B_R, $Pr_{min-max}$ as in the figure. 1), GENCOs need best values for bidding parameters in Energy as well as in Ancillary Service Markets, for their own and other participating generators. But, the solution obtained using Non-cooperative Nash Equilibrium Game Theory approach may not be optimal, as for any value of B_{ij}, P_{i1-3}, B_R, $Pr_{min-max}$ there exists a solution with a unit commitment schedule which may not be a global solution.This was explained in the following example.

Consider two GENCOs GC_1 and GC_2 and their bidding strategies as $B_{11}, B_{12}, B_{13}, B_{1R}$, for GC_1 and $B_{21}, B_{22}, B_{23}, B_{2R}$ for GC_2 respectively the model pay-off matrix is given in Eq. (18).

$$\left\{\begin{array}{ccc|cccccccc} & & & \multicolumn{4}{c}{GC_1} & \multicolumn{4}{c}{GC_2} \\ & & & B_{11} & B_{12} & B_{13} & B_{1R} & B_{21} & B_{22} & B_{23} & B_{2R} \\ GC_1 & & B_{11} & - & - & - & - & - & - & - & - \\ & & B_{12} & - & - & - & - & - & - & - & - \\ & & B_{13} & - & - & - & - & - & - & - & - \\ & & B_{1R} & - & - & - & - & - & - & - & - \\ GC_2 & & B_{21} & - & - & - & - & - & - & - & - \\ & & B_{22} & - & - & - & - & - & - & - & - \\ & & B_{23} & - & - & - & - & - & - & - & - \\ & & B_{2R} & - & - & - & - & - & - & - & - \end{array}\right\} \quad (18)$$

In the above matrix if the bidding strategies of GC_1 and GC_2 are same then there is a chance that two or more Nash Equilibriums may present and for each such strategy there exists an optimal unit commitment solution (Eq. 8), but not a global optimum solution. In this case, strong Nash Equilibrium can be obtained by observing strategies that are influencing other strategies, this process of finding stable strategy Nash Equilibrium can be termed as Evolutionary Game Theory (EGT). System Operator has to compute pay-off matrix for each hour and for each combination of unit commitment for finding the optimal solution; this increases the time of convergence for the problem considered in this paper.

EGT was proposed by John Maynard Smith and G.R. Price [20], which is initially used in Evolutionary Biology. It deals with the natural evolution of genes in the full population. It majorly depends on Darwin's theory of survival of the fittest. In the energy market, the players who survive in the repetitive competition only gives an optimal solution or in this case a Nash Equilibrium solution.

Evolutionary Game Theory is best suited for GENCOs bidding strategies but leads to slow convergence due to a large number of variables. So, a Hybrid method involving Evolutionary Game Theory with Differential Evolution (DE) [21] is proposed in this paper. DE does not require fine tuning of Mutation rate and cross-over parameters.

In game theory, the pay-off to each player bidding strategy is made depending on the decisions made by the entire market participant. All players can simultaneously reason the decisions made by other participants in the market. Convergence in game theory mainly depends on the availability of players information. On the other hand, Evolutionary Game Theory (EGT) analyses the explicit decisions made by the participants that can sustain in the market for a greater period. EGT is influenced by the frequency of the competing strategies that appear in the market [22].

This property of EGT is most suitable for the unit commitment based bidding strategy in Ancillary Service market as the players can foresee their chances of getting paid by altering their schedules.

The Hybrid Evolutionary Game Theory and Differential Evolution approach is explained in the following algorithm.

1) *Read the system data (population size, mutation factor, crossover rate, number of generators, cost coefficients, maximum and minimum power outputs, etc.)*

2) *Randomly fix the bid for each of the three blocks for each GENCO and find the marginal cost of bidding.*
3) *Run the Unit Commitment using Lagrangian Relaxation (LR) Method to obtain the power dispatched and the market clearing is done by using Economic Dispatch to obtain MCP.*
4) *Calculate the profits for each GENCO.*
5) *Take the first GENCO and find the maximum profit obtained by it by fixing the bidding strategies (Eq. (3)) of other GENCOs with the help of Differential Evolution.*
   I. *Randomly generate population 'pop' and determine their position 'd' in the population.*
   II. *Repeat following steps until the stopping criteria is reached.*
      a. *For each parent 'p' in 'pop' randomly select three other parents x,y,z*
      b. *Pick a random index I {1,2,…N}*
      c. *Compute new position 'd` for each of the parents*
   III. *For each parent pick a uniformly distributed number I_i= U (0, 1).*
   IV. *If I_i<Crossover Probability then set $Y_i=x_i+F(y_i - z_i)$. else set $Y_i=p_i$.*
   V. *If F(Y)>F(p), then replace that parent with a new child.*
   VI. *The best fit gives Best value of the objective function in the population.*
6) *Step 5 is repeated for all the GENCOs.*
7) *The strategy of each GENCO is updated.*
8) *Step (5-7) is repeated until no GENCO change its strategy.*
9) *The problem converges to Nash Equilibrium solution.*
10) *STOP.*

The flow chart of the method is given in Fig.3

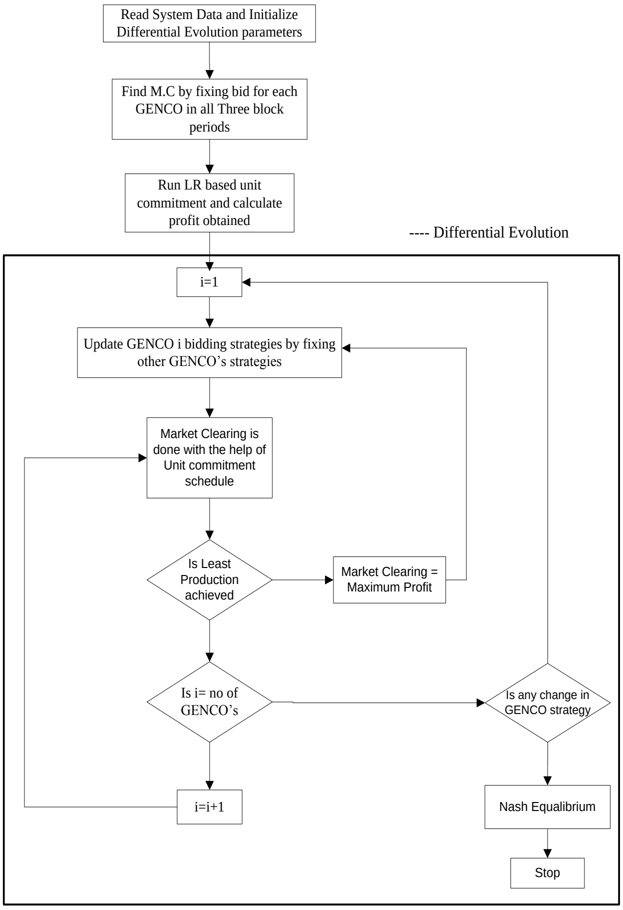

Fig. 3. Flow chart

5. Case Study

The WSCC 9 bus system (Fig. 4) and New England 39 bus Systems (Fig. 7) are used to demonstrate the proposed method. MATLAB coding for given algorithm run on Intel i5 processor with 4GB of RAM.

5.1 Case Study: WSCC 9 bus

The WSCC 9 bus system has 3 Generators whose cost characteristics are given in Table 1. These three generators are considered into two GENCOs. A Total reserve requirement of 10% and Probability of calling a reserve r=0.5[4] is considered. The payoff matrix for the bidding strategy of GENCOs is formed by considering 1.5 times its marginal cost. The maximum price paid to a reserve is considered be 110$/MW [5]. Table 2 provides load data.

In 9 bus case Each GENCO has 4 Bidding strategies and for each hour the unit commitment schedule has three factorial combinations. The size of pay-off matrix is huge in Game Theory for finding bidding strategies. To overcome this EGT method along with DE is proposed in this paper.

Table 1. Cost coefficients of generators

UNIT	a ($/MW²hr)	b ($/MWhr)	c ($/hr)	Pg_{min} (MW)	Pg_{max} (MW)
1	0.11	5	150	60	250
2	0.085	1.2	600	100	300
3	0.1225	1	335	60	270

Table 2. Load Data

Hour	Load	Hour	Load	Hour	Load	Hour	Load
1	545	7	436	13	658	19	662
2	517	8	456	14	673	20	657
3	491	9	506	15	659	21	642
4	469	10	561	16	640	22	641
5	456	11	607	17	628	23	633
6	458	12	639	18	648	24	568

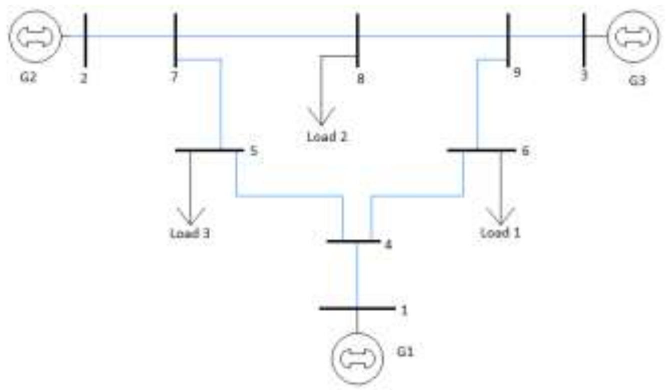

Fig. 4. WSCC 9 bus system

Case 1: In this case, GENCOs bidding strategies are considered at marginal cost (M.C) in Energy market. Operating Reserve in Ancillary Service market is not considered. The Optimal bidding strategies with the unit schedule in Day-ahead market are obtained. Table 3 shows the Unit commitment of 3 units along with generation schedule. Expected profit to the GENCOs in the energy market tabulated in Table 4. The shape of DE vector for case 1 is given in figure 5.

B_{11}	B_{12}	B_{13}	B_{21}	B_{22}	B_{23}	B_{31}	B_{32}	B_{33}
Pg_{11}	Pg_{12}	Pg_{13}	Pg_{21}	Pg_{22}	Pg_{23}	Pg_{31}	Pg_{32}	Pg_{33}

Fig. 5. DE Vector for case 1

DE parameters used:
Population size: 40,
Mutation rate: 0.5,
Crossover rate: 0.8

Table 3. 3 generator unit commitment with schedule in case 1

Hrs	Unit 1	Unit 3	Unit 2
	GENCO 1		GENCO 2
1	158.6	158.8	227.6
2	149.8	150.9	216.3
3	141.7	143.6	205.7
4	134.8	137.4	196.8
5	130.7	133.7	191.5
6	131.4	134.3	192.4
7	124.5	128.1	183.4
8	130.7	133.7	191.5
9	146.4	147.8	211.8
10	163.6	163.3	234.1
11	178	176.2	252.8
12	188.1	185.2	265.7
13	194	190.5	273.4
14	198.7	194.8	279.5
15	194.3	190.8	273.8
16	188.4	185.5	266.1
17	184.6	182.1	261.3
18	190.9	187.7	269.4
19	195.3	191.7	275.1
20	193.7	190.3	273
21	189	186	266.9
22	188.7	185.8	266.5
23	186.2	183.5	263.3
24	165.8	165.2	236.9
Total Operating Cost (Production Cost (P.C))			3,38,967.5 $

Table 4. Profit for each genco in Energy Market (EM)

Market	GENCO 1	GENCO2
Energy	72145$	72458$

Case 2: Along with above case 1 Optimal Bidding of GENCOs at marginal cost (M.C) in Energy market and max cap price for Operating Reserve in Ancillary Service Markets are considered, without network constraints. This case 2 has a significant effect on unit commitment schedule. The Unit commitment along with generation schedule of 3 generators is given in Table 5. And the amount allocated for each GENCO in Energy and Ancillary Service Markets is provided in Table 6. The shape of DE vector for case 2 is given figure 6.

B_{11}	B_{12}	B_{13}	B_{R1}	B_{21}	B_{22}	B_{23}	B_{R2}	B_{31}	B_{32}	B_{33}	B_{R3}
Pg_{11}	Pg_{12}	Pg_{13}	PR_1	Pg_{21}	Pg_{22}	Pg_{23}	PR_2	Pg_{31}	Pg_{32}	Pg_{33}	PR_3

Fig. 6. DE Vector for case 2

Table 5. Three generator Unit commitment with schedule in case 2

Hrs	Unit 1	Unit 3	Unit 2
	GENCO 1		GENCO 2
1	175.8	174.2	249.9
2	166.1	165.5	237.4
3	157.1	157.4	225.6
4	149.5	150.6	215.9
5	145.1	146.7	210.2
6	145.8	147.2	211
7	138.3	140.5	201.3

8	145.1	146.7	210.2
9	162.4	162.1	232.5
10	181.2	179	256.8
11	197.2	193.4	277.5
12	208.1	203.2	291.7
13	214.8	209.2	300
14	223.2	216.8	300
15	215.3	209.7	300
16	208.4	203.5	292.1
17	204.4	199.8	286.8
18	211.2	206	295.7
19	216.9	211.1	300
20	214.4	208.8	299.8
21	209.1	204	292.9
22	208.7	203.8	292.5
23	205.9	201.2	288.8
24	183.7	181.3	260.1
TOTAL OPERATING COST(P.C+R.C)			401298.8 $
TOTAL RESERVE COST Reserve Cost (R.C)			62331.3$

Table 6. Profit for each genco in Energy Market (EM) and Ancillary service Market (ASM)

Market	GENCO 1	GENCO2
Energy	55487$	53524$
Ancillary Service	15875$	18220$

Case 3: Along with case 1 and 2 the probability of calling reserve is considered. Which is considered to be 0.5 (50%)(r). Along with the case 2, network constraints are considered. The unit commitment of 3 generators with their generation schedule is given in Table 7 and profit for the both the markets is shown in Table 8.

Table 7. 3 generator Unit commitment and generation schedule in case 3

Hrs	Unit 1	Unit 3	Unit 2
	GENCO 1		GENCO 2
1	158.6	158.8	227.6
2	166.1	165.5	237.4
3	141.7	143.6	205.7
4	134.8	137.4	196.8
5	145.1	146.7	210.2
6	145.8	147.2	211
7	124.5	128.1	183.4
8	130.7	133.7	191.5
9	162.4	162.1	232.5
10	163.6	163.3	234.1
11	178	176.2	252.8
12	208.1	203.2	291.7
13	214.8	209.2	300
14	223.2	216.8	300
15	194.3	190.8	273.8
16	208.4	203.5	292.1
17	184.6	182.1	261.3
18	211.2	206	295.7
19	195.3	191.7	275.1
20	214.4	208.8	299.8
21	209.1	204	292.9
22	208.7	203.8	292.5
23	186.2	183.5	263.3
24	165.8	165.2	236.9
TOTAL OPERATING COST(P.C+R.C)			371958.8 $
TOTAL RESERVE COST (R.C)			32991.3 $

Table 8. Profit for each genco in Energy (EM) and Ancillary service Market (ASM)

Market	GENCO 1	GENCO2
Energy	55478$	57824$
Ancillary Service	8745$	7858$

The problem considered in Eq. (17) is solved with the algorithm proposed in section 4 and results tabulated in Tables 4, 6, 8.

The profit comparisons for various case studies done for WSCC 9 bus system are given in Fig. 7 and Fig. 8. It is clearly evident that Reserve market influences the Energy market's profit.

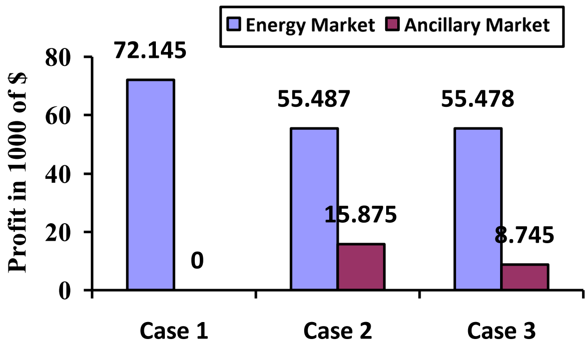

Fig. 7. Profit comparison of GENCO 1 in various cases

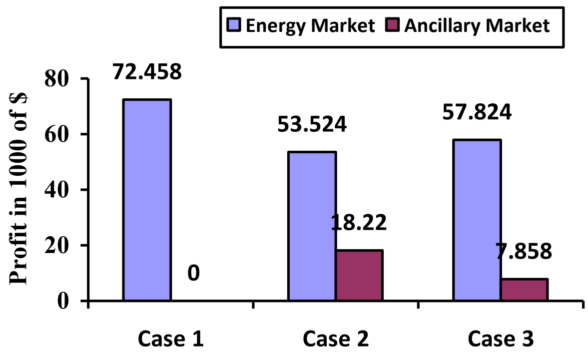

Fig. 8. Profit comparison of GENCO 2 in various cases

5.2 Case Study: New England 39 bus

The New England 39 bus System (Figure. 9) has 10 Generators, whose cost characteristics are given in Table 9. These ten generators are grouped into three GENCOs (Table 11) based on ownership [23]. Table 10 provides load data. In 39 bus case Each GENCO has 4 Bidding strategies and for each hour the unit commitment schedule has ten factorial combinations. This gives a huge payoff matrix in normal Game Theory for forming bidding strategies.

Table 9. Cost coefficients of generators

GENCO	UNIT	a ($/MW^{2}hr)	b ($/MWhr)	c ($/hr)	Pg_{min} (MW)	Pg_{max} (MW)
1	31	0.00150	7.65	459	60	1145
	32	0.00197	7.17	459	37.5	750
	39	0.00240	6.12	414	60	1100
2	30	0.00990	6.30	510	10	350
	37	0.00250	6.93	389	32	640
3	33	0.00165	6.88	619	40	732
	34	0.00145	7.52	561	30	608
	35	0.00272	7.09	489	37.5	750
	36	0.00610	5.30	589	33	660
	38	0.00210	7.04	347	45	930

Table 10. Load data

Hour	Load	Hour	Load	Hour	Load	Hour	Load
1	4200	7	5040	13	6658	19	6960
2	4432	8	5458	14	6784	20	6544
3	4558	9	5746	15	6922	21	6174
4	4746	10	5998	16	7224	22	5124
5	4852	11	6286	17	7476	23	4558
6	4936	12	6532	18	7596	24	4116

Table 11. Generators' grouping

GENCO	Generators
1	31,32,39
2	30,37
3	33,34,35,36,38

Case 1: Here GENCOs bidding strategies are considered at marginal cost (M.C) in Energy market. Operating Reserve in Ancillary Service market is not considered. The Optimal bidding strategies with the unit schedule in Day-ahead market are obtained. Table 12 shows the Unit commitment of 10 units. Expected profit to the GENCOs in the energy market is tabulated in Table 13.

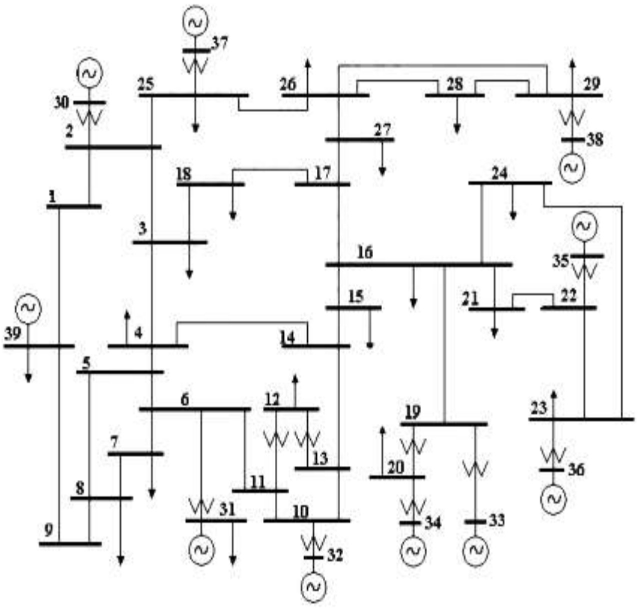

Fig. 9. New England 39 bus system

DE parameters used:
Population size: 40,
Mutation rate: 0.5,
Crossover rate: 0.8

Table 12. 10 generator Unit commitment in case 1

Unit	Hour (1-24)																						
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
7	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
8	0	0	0	0	0	0	1	1	0	1	1	1	1	0	0	1	1	1	1	1	1	0	1	0
9	0	0	0	0	0	0	0	0	1	1	1	1	0	1	1	1	1	1	1	1	1	1	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	0	0	0	0	0

Table 13. Profit for each genco in Energy (EM) and Ancillary service Market (ASM)

Market	GENCO 1	GENCO2	GENCO3
Energy	820511$	929118$	753094$

Case 2: Along with above case 1 Optimal Bidding of GENCOs at marginal cost (M.C) in Energy market and max cap price for Operating Reserve in Ancillary Service Markets are considered, without network constraints. This case 2 has a significant effect on unit commitment schedule. The Unit commitment of 10 generators is given in Table 14. And the profit for each GENCO in Energy and Ancillary service markets is given in Table 15.

Table 14. 10 generator Unit commitment in case 2

Unit	Hour (1-24)																						
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
7	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
8	0	0	0	0	0	0	0	1	1	0	0	1	1	1	1	1	1	1	1	1	0	1	0	0
9	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0

Table 15. Profit for each genco in Energy (EM) and Ancillary service Market (ASM)

Market	GENCO 1	GENCO2	GENCO3
Energy	530511$	761118$	533094$
Ancillary Service	122444$	68220$	120256$

Case 3: Along with case 1 and 2 the probability of calling reserve is considered. Which is considered to be 0.5 (50%)(r). Along with the case 2, Network constraints are considered. The unit commitment of 10 generators given in Table 16 and Profit for the both the market is provided in table 17.

Table 16. 10 generator Unit commitment in case 3

Unit	Hour (1-24)																							
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
6	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
7	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
8	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0

Table 17. Profit for each genco in Energy (EM) and Ancillary service Market (ASM)

Market	GENCO 1	GENCO2	GENCO3
Energy	540525$	842251$	582584$
Ancillary Service	61222$	34110$	60128$

The problem considered in Eq. (17) is solved with the algorithm proposed in section 4 and results tabulated in Tables 13, 15, 17.

The profit comparisons for various case studies done for New England 39 bus system are given in Fig. 10, 11 and Fig. 12. It is clearly evident that Reserve market influences the Energy market's profit.

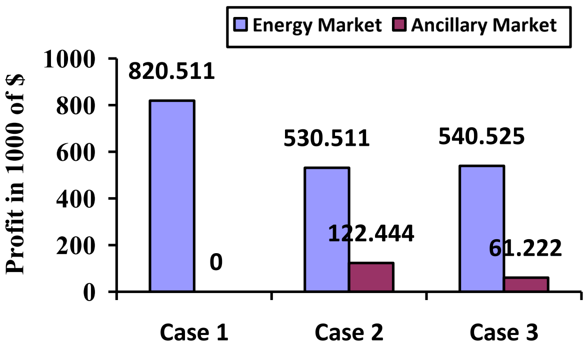

Fig. 10. Profit comparison of GENCO 1 in various cases

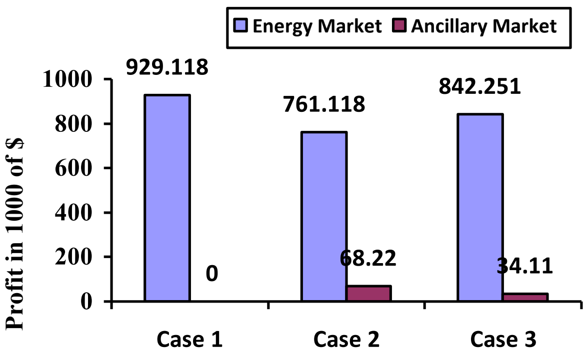

Fig. 11. Profit comparison of GENCO 2 in various cases

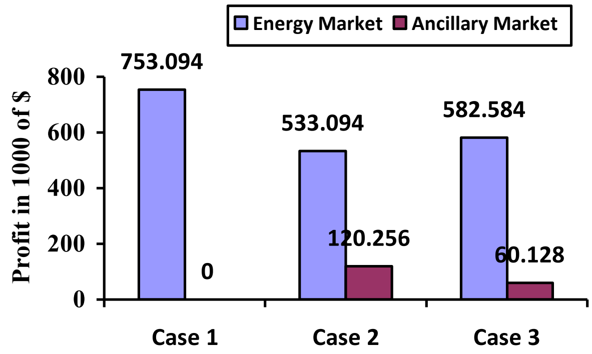

Fig.12. Profit comparison of GENCO 3 in various cases

6. Comparison Results

The convergence characteristics of the proposed method are compared with Game theory approach and Differential Evolution approach. Figure 13 gives the comparison of profit obtained by the GENCO 2 in New England 39 Bus system versus iterations. The values are tabulated in the Table 18.

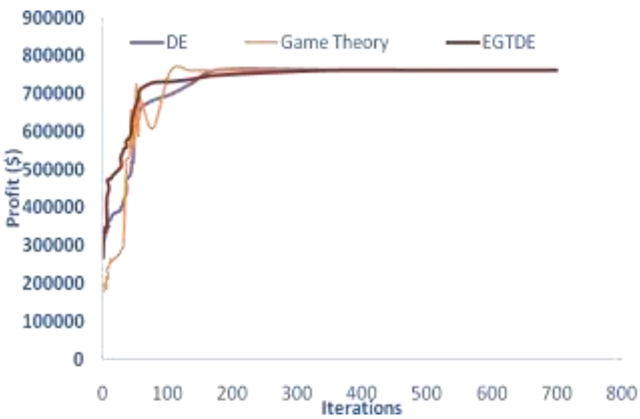

Fig. 13. comparison of profit vs no of iterations for GENCO-2 in New England 39 Bus system

Table 18. Number of iterations and Time taken for convergence

Method	Number of Iterations for Maximum profit	Time taken for convergence (in sec)
Game Theory	348	300
Differential Evolution	176	127
Proposed Method	105	36

From above Table 18 it is evident that the proposed method out performed both Game theory and Differential Evolution methods. The proposed method allows the system operator to schedule the ancillary service requirement appropriately and thus help in maintaining system balance.

7. Conclusion

A hybrid Evolutionary Game Theory based Differential Evolution approach is proposed in this paper for optimal bidding of GENCOs in Energy and Ancillary Services Markets. The proposed approach uses Supply Function Equilibrium (SFE) model for GENCOs with incomplete information. Committing the units as both Energy and Ancillary Services providers in the day-ahead market is a more realistic problem in the power system. Therefore, GENCOs try to maximize their profit by bidding rationally, by contemplating the characteristics of the other generators with minimally accessible information. Proposed optimization approach in this paper is a novel one, which gives optimal unit commitment and bidding strategies for GENCOs in both Energy and Ancillary Service Markets. The proposed method works well, attains faster convergence and optimal profit in 24-hour day-ahead Energy and Ancillary Service Markets over the Game Theory approach which is supported in the results section. As the future scope of the work, minimum emission objective function along with the cost function and the double sided auctions with mixed strategies can be considered.

Appendix

A1.1. Line Data for 9 Bus WSCC system

Bus No		Reactance (p.u)	Resistance (p.u.)
From Bus	To Bus		
1	4	0.0576	0.0
4	6	0.0920	0.017
3	9	0.0586	0.0
6	9	0.1700	0.039
5	7	0.1610	0.032
7	8	0.0720	0.0085
2	7	0.0625	0.0
8	9	0.1008	0.0119

A1.2. Line Data for 39 Bus New England system

Bus No		Reactance (p.u)	Resistance (p.u.)
From Bus	To Bus		
1	2	0.0411	0.00411
1			
2			
2			
3			
3			
4			
4			
5			
5			
6			
6			
7			
8			
9			
10			
10	13	0.0043	0.00043
13			
14			
15			
16			
16			
16			
16			
17			
17			
21			
22			
23			
25			
26			
26			
26			
28			
12			
12			
6			
10			
19			
20			
22			
23			
25			
2			
29			
19	20	0.0138	0

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