Risk-Aware Control Approach for Decision-Making System of a Shared EVs Aggregator

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Abstract- This paper proposes a risk-aware control approach intended to generate the most profitable decisions for the manager of a public fleet of electric vehicles that can interact bidirectionally with the electrical network, providing different energy services to it. Specifically, the proposed control approach is intended to generate the best charging/discharging decisions for the fleet, including car-sharing uncertainties and the desired confidence level at which the fleet operator wants to cover these uncertainties. It considers a hierarchical control structure at whose first level an economic dynamic optimization is executed, and, at whose second level, a risk-aware reference tracking of the first-level references is performed. Using this a stochastic MPC controller at the second level, whose mathematical approach has as a novelty that it extends the current methodologies in the state of the art, allowing the inclusion of the linear time-varying behavior of the dynamic system, whose constraints are also time-varying, and whose uncertainties are additive-multiplicative with non-zero mean and non-unitary variance. Finally, the approach established is tested in a hypothetical car-sharing system located in Colombia.

Keywords- Aggregator, Car-sharing, Decision-Making System, Electric Vehicles, Risk-aware Control, Stochastic control.

1 Introduction

Since 2014, the topic that addresses control strategies for electric vehicles (EVs) that bi-directionally exchange power with the electrical (V2G mode), has been gaining relevance, as the EVs can be used flexible storage elements when they are connected to the charging facilities. Hence, in those moments, the EVs can provide different kind of services to the power network, and they can sell energy. These new sources of income can help to improve the profitability of the EVs project, even more, when these are intended to supply a public transport demand as in the case of the car-sharing schemes. Because one of the most representative barriers to the implementation of this kind of project is the low-income margins for the aggregators of public or shared EVS.

Delving in such matter, in the state of the art, there are two main approaches to address the EVs aggregator management problem: the direct [1] and the hierarchical [2]. Specifically, in the direct approach, the solution for the problem is found in one stage, in which different control approaches such as, dynamic optimization [3]–[5] and Economic Model Predictive Control (EMPC) [6], [7], are used. However, the direct approach requires guaranteeing the stability of the proposed feedback law which can become a complex task if the system is nonlinear or if the objective function of the problem is non-convex [2]. This fact has motivated the choice of hierarchical techniques to solve the problem of economic control, as the authors specify in [2].

Wherefore, considering the features and drawbacks of the direct control approach, this paper uses the hierarchical scheme to calculate the optimal charging/discharging decisions for the operator of a shared EVs fleet (also called aggregator) who participates in the electricity sales and ancillary services provision markets.

More specifically, this second level is in charge of the EVs power and energy re-scheduling, adapting them to the new real conditions of the system, which can deviate from the mean expected conditions, considered at the first control level [8].

To conduct the tasks of this second level, there exists a variety of methods in the current state of the art. They are classified in the next categories, which are related to the uncertainties treatment: deterministic, such as the presented in [9], [10], and stochastic such as those presented in [11], [12].

Moreover, the last category that treats the uncertainties in a stochastic way, is part of a global topic that addresses stochastic processes and control methods topic is composed of the following three main subcategories.

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The robust approach, which characterizes the stochastic variables as sets with known bounds and calculates the control actions looking for the constraints fulfillment for all the elements in these sets [13]–[15].

The stochastic equations approach that models the system dynamics as a stochastic process and includes the desired risk level in the cost function or constraints of the controller [16]–[19].

And the Montecarlo approach, relies on a sampling process to derive the probability density function of the aleatory variables [11], [20]–[22].

Altogether, this paper includes the uncertainties (multiplicative and additive) at the second level within stochastic approach, having this feature as main difference from the approaches followed in references [23], [24], where robust controllers were proposed. This implies that the controllers set in [23], [24] are not able to include the probability density function of the uncertainties explicitly, as they are modeled as polytopes. Therefore, in the robust approach, the risk that the uncertainties deviate from their mean value, cannot be estimated directly, hence, it cannot be considered in the cost function and neither restricted in the constraints of the system. Besides, under the adopted stochastic control approach, the probability density function of the system uncertainties is included and restricted directly in the constraints, following the mathematical procedure proposed in reference [19], but extending it for linear time-varying systems under non-zero mean and non-unitary variance disturbances with time-varying disturbances, making the risk-aware stochastic MPC approach applicable for the car-sharing system model adopted. Additionally, the selected risk-aware stochastic approach avoids the accuracy and computational drawbacks derived from the Montecarlo methods.

In the remainder of the paper, the control architecture is depicted in Section 2, whereas the uncertainties characterization is presented in Section 3. Furthermore, the risk-aware control approach is detailed in Section 4. On the other hand, Section 5 describes the obtained with the application of the proposed controller to the selected car-sharing study case. Finally, Section 6 depicts the main highlights of the paper and the future research directions for the improvements of the results.

2. The Risk-Aware Control Approach

1.1 General Description of the Dynamic System

The system under consideration in this study is a station-based car-sharing, in which the EVs flow among the pre-defined charging stations to supply the transport requirements. In addition to this, the vehicles parked at the charging stations can sell energy and provide frequency regulation services to the electrical power network.

Regarding the frequency regulation services, there are two ways in which they can be provided: a) the upward regulation, and b) the downward regulation. For both services, the Electrical System Operator requires that the EVs aggregator defines, in a Day-Ahead (DA) horizon, the capacity or reserve that the EVs will assign for providing the upward and downward regulation. These capacities given for each hour of the day are denoted by P_{rd} and P_{ru}.

A complete dynamic description of the aforementioned car-sharing system model is given in the previous work [25], which corresponds with an aggregated energy model. This aggregated energy model is similar to the one proposed in [8]. However, the model used in [25] and in the current paper includes the features of the ancillary and car-sharing transport service provision. Particularly, in [25] the mentioned aggregated energy model for the shared EVs fleet is employed to estimate the optimal hourly DA scheduling of the ancillary services provision, the energy sales, and energy purchases for the whole fleet of EVs.

Although, these results were obtained under the assumption that the number of vehicles performing a trip among the stations (N_v), and the energy consumed during those trips (E_c) behaved as their expected values. Neglecting the fact that these two variables are stochastic. Impacting with this the accuracy of the estimated incomes from the car-sharing transport service. Hence, this paper, considers the hierarchical control scheme illustrated in Fig 1, which considers the case in which the variables N_v and E_c have a deviation regarding the mean condition.

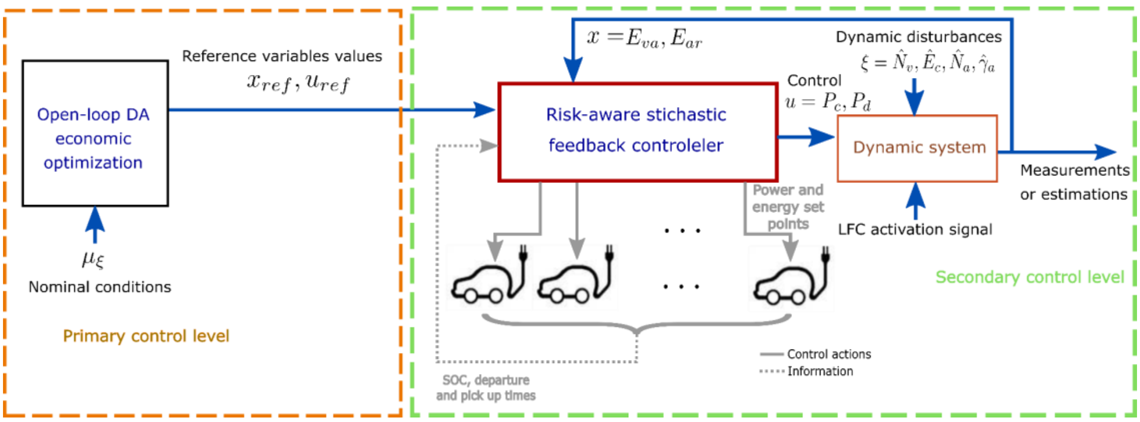


Fig 1. Scheme of the proposed hierarchical control approach.

On Fig 1, two control levels are set: The primary control level, in charge of generating the economic optimum reference values of the system, states $\boldsymbol{x_{ref}}$ and the inputs $\boldsymbol{u_{ref}}$. This level is fed by the mean condition of the car-sharing system disturbances $\boldsymbol{\mu_\xi}$, and the secondary frequency regulation activation signal (named in Fig 1 as LFC activation signal). And the secondary control level, which oversees regulating the changes in the charging/discharging processes and in the battery's energy level considering the uncertain behavior of the stochastic variables ($\hat{N}_v, \hat{E}_c, \hat{N}_a, \hat{\gamma}_a$) and the desired risk level at which the system should compensate the uncertainties. Moreover, this secondary level controller should be in charge of splitting the total power and energy of the aggregated vehicle´s set into the individual vehicles that are connected at the charging stations, based on the measured state of charge of the vehicles (SOC) and their departure schedules. However, this task is considered out of the scope of the paper and is not addressed.

1.2 First Control Level

The mathematical formulation of the first level control stage is given in Eqn. (1). It corresponds with an economic control problem which is deeply described in [23].

$$\min_{\boldsymbol{u_{ref}}} \sum_{k=1}^{N} g_1\left(\boldsymbol{x_{ref}}(k), \boldsymbol{u_{ref}}(k), \boldsymbol{\mu_\xi}(k)\right) \quad (1.1)$$

Subject to:

$$\boldsymbol{x_{ref}}(k+1) = \boldsymbol{f}\left(\boldsymbol{x_{ref}}(k), \boldsymbol{u_{ref}}(k), \boldsymbol{\mu_\xi}(k)\right), \quad (1.2)$$

$$\boldsymbol{h}\left(\boldsymbol{x_{ref}}(k), \boldsymbol{u_{ref}}(k), \boldsymbol{\mu_\xi}(k)\right) \leq 0; \quad (1.3)$$

$$\boldsymbol{x_{ref}}(N+1) \in X_f, \quad k \in \{1, \ldots, N\} \quad (1.4)$$

where $g_1(\cdot)$ is a function that estimates aggregator's cash-flow on a daily basis, $\boldsymbol{f}(\cdot)$ is the state transition function of the model that represents the car-sharing system dynamics, $\boldsymbol{h}(\cdot)$ is the function associated to the system constraints for energy and power limits, and X_f is a defined terminal set that guarantees the stability of the control.

On the other hand, the reference inputs $\boldsymbol{u_{ref}}$, the reference states $\boldsymbol{x_{ref}}$, and the char-sharing system disturbances $\boldsymbol{\xi}$ are:

$$\boldsymbol{u_{ref}}(k) = [P_{c,ref}(k) \quad P_{d,ref}(k)]^T; \quad (2.1)$$

$$\boldsymbol{x_{ref}}(k) = \left[\hat{E}_{b,ref}(k) \quad \hat{E}_{a,ref}(k)\right]^T; \quad (2.2)$$

$$\boldsymbol{\xi}(k) = [\hat{N}_v(k) \quad \hat{E}_c(k)]^T \quad (2.3)$$

being $P_{c,ref}(k)$ and $P_{d,ref}(k)$ the power charged/discharged in the time slot $[k, k+1)$, respectively; $\hat{E}_{b,ref}(k)$ the energy stored in all the batteries of the vehicles parked charging stations at the time step k which have arrived at least since $k-1$, and $\hat{E}_{a,ref}(k)$ is the remaining energy content in the batteries of vehicles that have arrived to the charging stations in the time slot $[k, k+1)$ that stay until $k+1$.

And finally, the uncertain variables $\boldsymbol{\xi}$ are the vehicles making trips among the stations $\hat{N}_v(k)$ in the time slot $[k, k+1)$, and the energy consumed by the EVs departing from the time step k and arriving at the time step $k+1$ denoted as $\hat{E}_c(k)$.

1.3 Second Control Level

Besides, the second control level relies on a state feedback law, which minimizes the difference between new calculated input and state variables, considering the system uncertainties, regarding their reference values. These deviation variables are calculated as $\Delta\boldsymbol{x}(k) = \boldsymbol{x}(k) - \boldsymbol{x_{ref}}(k)$, $\Delta\boldsymbol{u}(k) = \boldsymbol{u}(k) - \boldsymbol{u_{ref}}(k)$, and $\Delta\boldsymbol{\xi}(k) = \boldsymbol{\xi}(k) - \boldsymbol{\mu_\xi}(k)$.

The dynamic model that relates the deviation variables $\Delta\boldsymbol{x}$ and $\Delta\boldsymbol{u}$, is described in Eqn. (3).

$$\Delta\hat{E}_b(k+1) = [\Delta P_c(k)]\eta_c - [\Delta P_d(k)]\frac{1}{\eta_d} + \Delta\hat{E}_a(k) + \left(1 - \frac{\gamma_{c,ref}(k)\hat{N}_v(k)}{\hat{N}_p(k)}\right)\Delta\hat{E}_b(k); \quad (3.1)$$

$$\Delta\hat{E}_a(k+1) = \Delta\hat{E}_b(k)\frac{\hat{N}_v(k)}{\hat{N}_p(k)} - \gamma_{c,ref}(k)\Delta\hat{E}_c(k) \quad (3.2)$$

where η_c and η_d are the charging and discharging efficiencies of the EVs.

It is highlighted that the deviation variables $\Delta\hat{E}_b$, $\Delta\hat{E}_a$, ΔP_c, and ΔP_d are obtained considering that the values of the variables $\hat{E}_{b,ref}$, $\hat{E}_{a,ref}$, $P_{c,ref}$, $P_{d,ref}$, $P_{rd,ref}$, and $P_{ru,ref}$ are previously set at the first control level.

Furthermore, the linear simplified model of (3) was generated assuming that the car-sharing service price V_c and the ancillary services calculated reserves, P_{rd}, P_{ru}, remain constant at their optimal reference values $V_{c,ref}$, $P_{rd,ref}$, and $P_{ru,ref}$. Hence, they are taken as parameters coming from the first control level. Moreover, the portion of the total car-sharing demand covered $\gamma_{c,ref}$ is obtained with the expression set in (4) that relates the demand and the transport service price of the car-sharing system.

$$\gamma_{c,ref}(k) = p_1 \cdot e^{p_2 \cdot V_{c,ref}(k)} + p_3 \cdot e^{p_4 \cdot V_{c,ref}(k)} \quad (3)$$

being p_1, p_2, p_3, and p_4 parameters that depend on the car-sharing users preferences, calculated in [23].

Moreover, as it was indicated previously, the proposed approach considers two main uncertain variables in this model: the vehicles performing trips $\hat{N}_v$ and the total energy consumption during those trips $\hat{E}_c$; and other three secondary uncertain variables related to the main ones, which are the vehicles parked $\hat{N}_p$, the vehicles arriving $\hat{N}_a$, and the departure-available vehicles' ratio $\hat{\gamma}_a$.

$$\hat{N}_p(k+1) = \hat{N}_p(k) + \hat{N}_a(k) - \gamma_{c,ref}(k)\hat{N}_v(k); \quad (3.4)$$

$$\hat{\gamma}_a(k) = \frac{\gamma_{c,ref}(k) \cdot \hat{N}_v(k)}{\hat{N}_p(k)} \quad (3.5)$$

Where, (5.1) considers that the vehicles that depart at $k-1$ arrives at k-th time step. Hence, the number of EVs that arrive at the time step k, $\hat{N}_a(k)$, are calculated as $\hat{N}_a(k) = \gamma_{c,ref}(k-1)\hat{N}_v(k-1)$.

Therefore, the model in (3), can be rewritten in terms of these new uncertain variables as Eqn. (6) indicates:

$$\Delta\hat{E}_b(k+1) = [\Delta P_c(k)]\eta_c - [\Delta P_d(k)]\frac{1}{\eta_d} + \left(1 - \hat{\gamma}_a(k)\right)\Delta\hat{E}_b(k) + \Delta\hat{E}_a(k); \quad (4.1)$$

$$\Delta\hat{E}_a(k+1) = \hat{\gamma}_a(k) \cdot \hat{E}_a(k) - \Delta\hat{E}_p(k) \quad (4.2)$$

being $\hat{E}_p(k) = \gamma_{c,ref}(k)\hat{E}_c(k)$.

At this point, it should be noted that there are two kinds of uncertain variables in (6):

The uncertain variable additive with the states $\Delta\hat{E}_p(k)$ (since it is a subtracting term in (6.2). Whose expected value and variance $\mu_{\Delta\hat{E}_p}(k) = 0$, are obtained from the stochastic characterization of the variable $\hat{E}_p$, since $\Delta\hat{E}_p(k) = \hat{E}_p - \mu_{\Delta\hat{E}_p}$.

The uncertain variable multiplicative with the states $\hat{\gamma}_a(k)$ (since its multiplying to the state $\hat{E}_b$ in (6.1) and (6.2). Whose expected value and variance are $\mu_{\hat{\gamma}_a}$ and $\sigma^2_{\hat{\gamma}_a}$.

The procedure to obtain these values are detailed in Section 3.

Therefore, it is concluded that the model set in (6) belongs to the category of time-varying linear systems with additive and multiplicative disturbances, which can be represented in the general form given in Eqn. (7):

$$\begin{bmatrix}\Delta\boldsymbol{x}(k+1)\\ \boldsymbol{\eta}(k+1)\end{bmatrix} = \left(\begin{bmatrix}\boldsymbol{A_0}(k) & \boldsymbol{0}\\ \boldsymbol{CA_0}(k) & \boldsymbol{I}\end{bmatrix} + \begin{bmatrix}\boldsymbol{A_i}(k) & \boldsymbol{0}\\ \boldsymbol{CA_i}(k) & \boldsymbol{0}\end{bmatrix}\right)\begin{bmatrix}\Delta\boldsymbol{x}(k)\\ \boldsymbol{\eta}(k)\end{bmatrix} + \begin{bmatrix}\boldsymbol{B}\\ \boldsymbol{CB}\end{bmatrix}\Delta\boldsymbol{u}(k) + \begin{bmatrix}\boldsymbol{D}\\ \boldsymbol{CD}\end{bmatrix}\Delta\boldsymbol{\xi_a}(k) \quad (5)$$

The expression in (7) can be also represented in a compact form, as:

$$\widetilde{\boldsymbol{x}}(k+1) = \left(\widetilde{\boldsymbol{A_0}}(k) + \widetilde{\boldsymbol{A_\iota}}(k)\right)\widetilde{\boldsymbol{x}}(k) + \widetilde{\boldsymbol{B}}\Delta\boldsymbol{u}(k) + \widetilde{\boldsymbol{D}}\Delta\boldsymbol{\xi_a}(k) \quad (6)$$

where the variable $\Delta\boldsymbol{\xi_a}(k) = \Delta\hat{E}_p(k)$ denotes the additive uncertainties, while the multiplicative uncertainties are introduced in the matrices $\widetilde{\boldsymbol{A_0}}$ and $\widetilde{\boldsymbol{A_\iota}}$are given in (9).

Moreover, the states and inputs, set in (7), are $\widetilde{\boldsymbol{x}}(k)^T = [\Delta\hat{E}_b(k) \quad \Delta\hat{E}_a(k) \quad v_1 \quad v_2]$, and $\Delta\boldsymbol{u}(k)^T = [\Delta P_c(k) \quad \Delta P_d(k)]$. The augmented states v_1, v_2 are integrators of the errors for the states $\Delta\boldsymbol{x}$; and, since the states are measurable, the matrix $\boldsymbol{C}$ is equal to the identity.

$$\widetilde{\boldsymbol{A}}_{\boldsymbol{0}}(k) = \begin{bmatrix}1-\mu_{\hat{\gamma}_a}(k) & 1\\ \mu_{\hat{\gamma}_a}(k) & 0\end{bmatrix},\ \widetilde{\boldsymbol{A}}_{\boldsymbol{i}}(k) = \begin{bmatrix}-\sigma_{\hat{\gamma}_a}(k) & 0\\ \sigma_{\hat{\gamma}_a}(k) & 0\end{bmatrix} \quad (7)$$

3. Uncertainties Characterization

The mathematical approach of the complete system, formed by the model established in (6), and the constraints established (10), has 4 stochastic variables: $\Delta\hat{E}_p, \hat{\gamma}_a, \hat{N}_p, \hat{N}_a$, which need to be characterized.

Specifically, for the variable $\Delta\hat{E}_p$ it is necessary to determine its second moment $\sigma_{\hat{E}_p}$, (since its first moment $\mu_{\hat{E}_p} = 0$). And, as the variable $\hat{E}_p$ is the total energy consumption of vehicles traveling among the stations, it implies that this variable can be obtained from the product $\hat{E}_p = \gamma_{c,ref} \cdot \hat{N}_v(k) \cdot \hat{E}_v(k)$, were $\hat{N}_v(k)$ is the number of vehicles traveling and $\hat{E}_v(k)$ is the individual consumption of vehicles for a specific path. Therefore, the second moment of $\hat{E}_p$ can be derived as Eqn. (10) indicates:

$$Var\left[\hat{E}_p(k)\right] = \gamma^2_{c,ref}(k) \cdot Var\left[\sum_{s=1}^{D}\hat{N}_d(s,k)\sum_{s=1}^{D}\hat{E}_v(s,k)\right] \quad (8)$$

where $\hat{N}_d(s,k)$ is the number of vehicles traveling between some stations in the s-th direction at the k-th time step, and hence $\hat{N}_v(k) = \sum_{s=1}^{D}\hat{N}_d(s,k)$.

On the other hand, the variable $\hat{E}_v(s,k)$ is the consumed by an EV that takes the s-th path at the time slot $[k, k+1)$. Hence, the values for $\hat{N}_d(s,k)$ and $\hat{E}_v(s,k)$ are obtained from the traffic simulation of the considered study case network under different traffic scenarios and, and then, their first and second moments of these variables are estimated for deriving, posteriorly, the second moment of $\hat{E}_p$.

In an analogous way, the moments of the other stochastic variables are obtained. In this regard, the mean and variance of the variable $\hat{N}_a$ are calculated with expressions (11.1) and (11.2).

$$E\left[\hat{N}_a(k)\right] = \gamma_{c,ref}(k-1)\cdot E\left[\sum_{s=1}^{D}\widehat{N_d}(s,k-1)\right] = \gamma_{c,ref}(k-1)\cdot\sum_{s=1}^{D}\mu_{\hat{N}_d}(s,k-1); \quad (9.1)$$

$$Var\left[\hat{N}_a(k)\right] = \gamma^2_{c,ref}(k-1)\cdot E\left[\sum_{s=1}^{D}\hat{N}_d(s,k-1)\right] = \gamma^2_{c,ref}(k-1)\cdot\sum_{s=1}^{D}\sigma^2_{\hat{N}_d}(s,k-1) \quad (9.2)$$

Where the formulation given in Eqn. (11.2), considers that the samples of $\hat{N}_d$ are not correlated in time.

The first and second momentum of the variable $\hat{N}_p$ are calculated as Eqns. (12.1) and (12.2) indicate, which are based on the definition of $\hat{N}_p$ given in (5.1).

$$E\left[\hat{N}_p(k)\right] = \gamma_{c,ref}(k-1)\cdot E\left[N_t - \sum_{s=1}^{D}\hat{N}_d(s,k-1)\right] = \gamma_{c,ref}(k-1)\cdot\left(N_t - \sum_{s=1}^{D}\mu_{\hat{N}_d}(s,k-1)\right); \quad (10.1)$$

$$Var\left[\hat{N}_p(k)\right] = \gamma^2_{c,ref}(k-1)Var\left[N_t - \sum_{s=1}^{D}\hat{N}_d(s,k-1)\right] = \gamma^2_{c,ref}(k-1)\cdot\sum_{s=1}^{D}\sigma^2_{\hat{N}_d}(s,k-1);$$

(10.2)

where N_t is the fleet size.

Finally, the first and second moment of $\hat{\gamma}_a$ are obtained with (13.1) and (13.2), detailed described in [23].

$$E[\hat{\gamma}_a](k) = \frac{\gamma_{c,ref}(k)}{\gamma_{c,ref}(k-1)}E\left[\frac{\sum_{s=1}^{D}\hat{N}_d(s,k)}{N_t - \sum_{s=1}^{D}\hat{N}_d(s,k-1)}\right]; \quad (11.1)$$

$$Var[\hat{\gamma}_a(k)] = \frac{\gamma^2_{c,ref}(k)}{\gamma^2_{c,ref}(k-1)}Var\left[\frac{\left(\sum_{s=1}^{D}\hat{N}_d(s,k)\right)}{\hat{N}_p(k)}\right] \quad (11.2)$$

Now, considering the previous characterization of the first and second moment of the stochastic variables, they can be represented as stochastic variables as equation (14) indicates, assuming they are normally distributed:

$$\xi_1(k) = \Delta\hat{\gamma}_a(k) \sim \mathcal{N}\left(0, \quad \frac{\gamma_{c,ref}(k) \cdot \sigma_{\hat{\gamma}_a}(k)}{\gamma_{c,ref}(k-1)}\right); \tag{12.1}$$

$$\xi_2(k) = \Delta\hat{E}_p(k) \sim \mathcal{N}\left(0, \quad \gamma_{c,ref}(k) \cdot \sigma_{\hat{E}_c}(k)\right); \tag{12.2}$$

$$\xi_3(k) = \hat{N}_p(k) \sim \mathcal{N}\left(\gamma_{c,ref}(k-1) \cdot \left(N_t - \mu_{\hat{N}_v}(k-1)\right), \quad \gamma_{c,ref}(k-1) \cdot \sigma_{\hat{N}_v}(k-1)\right); \tag{12.3}$$

$$\xi_4(k) = \hat{N}_a(k) \sim \mathcal{N}\left(\gamma_{c,ref}(k-1) \cdot \mu_{\hat{N}_v}(k-1), \quad \gamma_{c,ref}(k-1) \cdot \sigma_{\hat{N}_v}(k-1)\right) \tag{12.4}$$

4. Stochastic Risk-Aware Controller Setting

This chapter describes the risk aware stochastic MPC approach followed to perform the second stage reference tracking task on Fig 1. Whose general mathematical approach is set in equation (15).

$$\min_{\Delta x, \Delta u} \left(\sum_{k=1}^{N} \parallel \Delta x(k) \parallel_Q^2 + \parallel \Delta u(k) \parallel_R^2 \right) +$$

$$\parallel \Delta x(N+1) \parallel_{P(N+1)}^2 + \left(\sum_{k=1}^{N} \mathrm{Tr}\left((Q + K(k)RK(k))\Sigma_{\Delta x}(k)\right)\right) + \mathrm{Tr}(P(N+1)\Sigma_{\Delta x}(N+1)) \tag{13.1}$$

$$Subject\ to:$$

$$\widetilde{\boldsymbol{x}}(k+1) = \left(\widetilde{\boldsymbol{A}_0}(k) + \widetilde{\boldsymbol{A}_i}(k)\right)\widetilde{\boldsymbol{x}}(k) + \widetilde{\boldsymbol{B}}\Delta\boldsymbol{u}(k) + \widetilde{\boldsymbol{D}}\Delta\boldsymbol{\xi_a}(k); \quad \forall k \in \{1, \ldots, N\}; \tag{13.2}$$

$$\mathbb{P}\left(\widetilde{\boldsymbol{A}_x} \cdot \begin{bmatrix} \widetilde{\boldsymbol{x}}(k) \\ \boldsymbol{\xi_c} \end{bmatrix} + \boldsymbol{b_x}(k) \le 0\right) \ge 1 - \epsilon_x, \quad \forall k \in \{1, \ldots, N\}; \tag{13.3}$$

$$\mathbb{P}\left(\boldsymbol{A_u} \cdot \begin{bmatrix} \Delta\boldsymbol{u}(k) \\ \boldsymbol{\xi_c} \end{bmatrix} + \boldsymbol{b_u}(k) \le 0\right) \ge 1 - \epsilon_u, \quad \forall k \in \{1, \ldots, N\}; \tag{13.4}$$

Where the variables ϵ_x and ϵ_u are the confidence levels for the states and inputs constraints accomplishment, respectively. These confidence levels include the risk, implicitly, as they guarantee the degree for constraint fulfillment and penalizes undesirable operational conditions. Furthermore, these variables can be included in the control objective function to economically penalize or reflect the cost of operating the system at certain points, using different penalizing functions, also called risk measures. But for the shake of simplicity, in this paper, the confidence values will be previously fixed.

Therefore, as the computation of constraints (15.3- 15.4) involves calculations over the probability density functions of the stochastic variables $\widetilde{\boldsymbol{x}}$ or operations with their moments, these constraints will be a joint-chanced constraints. Being $\boldsymbol{\xi_c} = [\xi_3 \quad \xi_4]^T$ the disturbances associated with the system constraints (see ref. [23] for the complete description of the constraints) which are the variables $\xi_3(k) = \hat{N}_p(k)$, and $\xi_4(k) = \hat{N}_a(k)$.

The mathematical approach set in equation (15) is similar to the control equation of a robust MPC set in [23] and [24]; however there are 2 main differences: the first one is the inclusion of a penalization term for the variance matrix of the states $\boldsymbol{\Sigma_{\Delta x}}$ in order to guarantee the mean-square stability of the stochastic process $\Delta\boldsymbol{x}$; and the second one is the addition of the joint chanced constraints in equations (15.3) - (15.4). On the other hand, in the work developed in [24] the uncertainties are treated as polytopes, without setting an specific relationship among the control law, the risk and the moments of the probability distribution of the uncertainties. Besides, the work presented reference [19], goes an step further and proposes a stochastic controller that takes into account the moments of the probability density function and the risk or confidence level of the uncertain variables to derive a control law for a discrete linear time invariant dynamic system for additive and multiplicative noises with zero mean and unitary variance. Thus, the present paper complements the work presented by [19], extending its mathematical approach for the linear time-varying system set in equation (15.2), whose uncertainties are non-zero mean, non-unitary variance, as can be observed in equation (14); which also has time-varying constraints, since the matrices $\boldsymbol{b_x}(k), \boldsymbol{b_u}(k)$ in (15.3) - (15.4) depend on the time step.

Specifically, equation (16) indicates the terms that compose the matrices in the constraints (15.3) - (15.4) which are derived from the system constraints set in [23]:

$$\widetilde{\boldsymbol{A}_x} = \begin{bmatrix} 1 & 0 & 0 & 0 & -E_M & 0 \\ 0 & 1 & 0 & 0 & 0 & -E_M \\ -1 & 0 & 0 & 0 & E_m & 0 \\ 0 & -1 & 0 & 0 & 0 & E_m \end{bmatrix};$$

$$\boldsymbol{A_u} = \begin{bmatrix} 1 & 0 & -P_M & 0 \\ 0 & 1 & -P_M & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}; \tag{14.1}$$

$$\boldsymbol{b_x}(k) = \begin{bmatrix} E_{b,ref}(k) - \eta_c P_{rd,ref}(k) \\ E_{a,ref}(k) - \eta_c P_{rd,ref}(k) \\ -E_{b,ref}(k) + \frac{1}{\eta_d} P_{ru,ref}(k) \\ -E_{a,ref}(k) + \frac{1}{\eta_d} P_{ru,ref}(k) \end{bmatrix};$$

$$\boldsymbol{b_u}(k) = \begin{bmatrix} P_{c,ref}(k) + P_{rd,ref}(k) \\ P_{d,ref}(k) + P_{ru,ref}(k) \\ 0 \\ 0 \end{bmatrix} \tag{14.2}$$

Where E_M is the maximum energy content allowed in the individual EVs, E_m is the minimum energy content allowed, and P_M is the maximum charging/discharging power.

Then, the joint chanced-constraints set in equations (15.3) - (15.4) are solved based on the Chebyshev bounds theory and Boole's inequality addressed in [26]; in which, these joint constraints are divided into individual constraints as follows:

$$\sqrt{\frac{1-\epsilon_{xi}}{\epsilon_{xi}}}\sqrt{\widetilde{\boldsymbol{a}}_{\boldsymbol{xi}}\begin{bmatrix}\boldsymbol{\Sigma}_{\widetilde{x}}(k) & cov(\widetilde{\boldsymbol{x}}(k),\boldsymbol{\xi_c}) \\ cov(\widetilde{\boldsymbol{x}}(k),\boldsymbol{\xi_c})^T & \boldsymbol{\Sigma}_{\xi_c}^T(k)\boldsymbol{\Sigma}_{\xi_c}(k)\end{bmatrix}\widetilde{\boldsymbol{a}}_{\boldsymbol{xi}}^T}+\widetilde{\boldsymbol{a}}_{\boldsymbol{xi}}\begin{bmatrix}\boldsymbol{\mu}_{\widetilde{x}}(k) \\ \boldsymbol{\mu}_{\xi_c}(k)\end{bmatrix}\le -b_{xi}(k),$$
$$\forall i\in\{1,\dots,N_x\},\quad \forall k\in\{1,\dots,N\}; \tag{15.1}$$

$$\sqrt{\frac{1-\epsilon_{ui}}{\epsilon_{ui}}}\sqrt{\boldsymbol{a}_{\boldsymbol{ui}}\begin{bmatrix}\boldsymbol{\Sigma}_{\Delta\boldsymbol{u}}(k) & cov(\Delta\boldsymbol{u}(k),\boldsymbol{\xi_c}) \\ cov(\Delta\boldsymbol{u}(k),\boldsymbol{\xi_c})^T & \boldsymbol{\Sigma}_{\xi_c}^T(k)\boldsymbol{\Sigma}_{\xi_c}(k)\end{bmatrix}\boldsymbol{a}_{\boldsymbol{ui}}^T}+\boldsymbol{a}_{\boldsymbol{ui}}\begin{bmatrix}\boldsymbol{\mu}_{\Delta\boldsymbol{u}}(k) \\ \boldsymbol{\mu}_{\xi_c}(k)\end{bmatrix}\le -b_{ui}(k),$$
$$\forall i\in\{1,\dots,N_u\},\quad \forall k\in\{1,\dots,N\} \tag{15.2}$$

Where $\widetilde{\boldsymbol{a}}_{\boldsymbol{xi}}, b_{xi}, \boldsymbol{a}_{\boldsymbol{ui}}$ and b_{ui} are row vectors and elements of the matrices $\widetilde{\boldsymbol{A}}_{\boldsymbol{x}}, \boldsymbol{b}_{\boldsymbol{x}}, \boldsymbol{A}_{\boldsymbol{u}}$ and $\boldsymbol{b}_{\boldsymbol{u}}$ respectively; and the term $f_s=\sqrt{\frac{1-\epsilon_i}{\epsilon_i}}$ indicates the multiplicative factor of the stochastic process standard deviation required to guarantee the selected risk level.

On the other hand, the control actions are given by the next expression:

$$\Delta\boldsymbol{u}(k)=\boldsymbol{K}(k)\widetilde{\boldsymbol{x}}(k)+\boldsymbol{c}(k) \tag{16}$$

Where the gain $\boldsymbol{K}(k)$ is estimated according to equation (22), and the dual variables $\boldsymbol{c}(k)$ are selected as eqn. (25) indicates.

Hence, the first moment of the variables $\widetilde{\boldsymbol{x}}$ and inputs $\Delta\boldsymbol{u}$ are calculated with Equations (19).

$$\boldsymbol{\mu}_{\Delta\boldsymbol{u}}(k)=\boldsymbol{K}(k)\cdot\boldsymbol{\mu}_{\widetilde{x}}(k)+\boldsymbol{c}(k);\quad \forall k\in\{1,\dots,N\}; \tag{17.1}$$

$$\boldsymbol{\Sigma}_{\Delta\boldsymbol{u}}(k)=\boldsymbol{K}(k)\cdot\boldsymbol{\Sigma}_{\widetilde{x}}(k)\cdot\boldsymbol{K}(k)^T \quad \forall k\in\{1,\dots,N\} \tag{17.2}$$

Outlining, the control setting consists of finding $\boldsymbol{K}(k)$ and $\boldsymbol{c}(k)$ that give the minimum reference tracking error (17), and that guarantee the accomplishment of the constraints. Furthermore, the control law must accomplish with the recursive Riccati Equation, and therefore the gain $\boldsymbol{K}(k)$ can be estimated as Equation (22) indicates, using Semidefinite Programming – SDP for the perturbed time-varying system in Eqn. (6) which described the evolution of the states, assuming some pre-selected values for the matrices $\boldsymbol{Q}$ and $\boldsymbol{R}$.

$$\left(\widetilde{\boldsymbol{A}}(k)+\widetilde{\boldsymbol{B}}\boldsymbol{K}(k)\right)^T\boldsymbol{P}(k+1)\widetilde{\boldsymbol{A}}(k)+\widetilde{\boldsymbol{B}}(k)\boldsymbol{K}(k))-\boldsymbol{P}(k)+\boldsymbol{Q}+\boldsymbol{K}(k)^T\boldsymbol{R}\boldsymbol{K}(k)\le 0 \tag{18}$$

Defining for this, the new auxiliary variables $\boldsymbol{S}\in\mathbb{R}^{n_x\times n_x\times(N+1)}$, $\boldsymbol{Y}\in\mathbb{R}^{n_u\times n_x\times N}$, and $\alpha\in\mathbb{R}$, relating them to the original ones as Equation (21) indicates.

$$\boldsymbol{K}(k)=\boldsymbol{y_k}\cdot\boldsymbol{s_k}^{-1},\ \boldsymbol{P}(k)=\alpha\ \boldsymbol{s_k}^{-1} \tag{19}$$

Then, the optimization can be reformulated as Eqn. (21) indicates:

$$\min_{S,Y,\alpha}\alpha \tag{20.1}$$
$$Subject\ to:$$
$$\alpha>0; \tag{20.2}$$
$$\begin{bmatrix}1 & \widetilde{\boldsymbol{x}}(1)^T \\ \widetilde{\boldsymbol{x}}(1) & s_{N+1}\end{bmatrix}\succcurlyeq 0; \tag{20.3}$$
$$\boldsymbol{s_k}=\boldsymbol{s_k}^T,\quad \forall k\in\{1,\dots,N+1\}; \tag{20.4}$$
$$s_k\succcurlyeq 0,\quad \forall k\in\{1,\dots,N+1\}; \tag{20.5}$$
$$\boldsymbol{s}_{N+1}=\alpha\cdot\boldsymbol{Q}^{-1}; \tag{20.6}$$
$$\begin{bmatrix}\boldsymbol{s_k} & (\widetilde{\boldsymbol{A}}_{\boldsymbol{0}}(k)+\widetilde{\boldsymbol{A}}_i(k))\cdot\boldsymbol{s_k}+\widetilde{\boldsymbol{B}}\cdot\boldsymbol{y_k})^T & \boldsymbol{s_k}\cdot\boldsymbol{Q}^{1/2} & \boldsymbol{y_k}\cdot\boldsymbol{R}^{1/2} \\ \widetilde{\boldsymbol{A}}_{\boldsymbol{0}}(k)+\widetilde{\boldsymbol{A}}_i(k))\cdot\boldsymbol{s_k}+\widetilde{\boldsymbol{B}}\cdot\boldsymbol{y_k} & \boldsymbol{s_{k+1}} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{Q}^{1/2}\cdot\boldsymbol{s_k} & \boldsymbol{0} & \alpha\boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{R}^{1/2}\cdot\boldsymbol{y_k} & \boldsymbol{0} & \boldsymbol{0} & \alpha\boldsymbol{I}\end{bmatrix}\succcurlyeq 0, \tag{20.7}$$
$$\forall k\in\{1,\dots,N\}$$

where $\boldsymbol{s_k}, \boldsymbol{y_k}$ are obtained as $\boldsymbol{s_k}=\boldsymbol{S}(\cdot,\cdot,k)$, $\boldsymbol{y_k}=\boldsymbol{Y}(\cdot,\cdot,k)$

On the other hand, as it was indicated previously, the matrix $\boldsymbol{\Sigma}_{\widetilde{x}}$ must be selected such that the mean square stability of the stochastic process $\widetilde{\boldsymbol{x}}$ holds. This can be guaranteed constraining $\boldsymbol{\Sigma}_{\widetilde{x}}$ to accomplish with the next Lyapunov Equation:

$$\widetilde{\boldsymbol{A}}_{\boldsymbol{0}}(k)+\widetilde{\boldsymbol{B}}\boldsymbol{K}(k))\cdot\boldsymbol{\Sigma}_{\widetilde{x}}(k)\cdot\left(\widetilde{\boldsymbol{A}}_{\boldsymbol{0}}(k)+\widetilde{\boldsymbol{B}}\boldsymbol{K}(k)\right)^T-\boldsymbol{\Sigma}_{\widetilde{x}}(k+1)+\overline{\boldsymbol{W}}(k)=\boldsymbol{0};\ \forall k\in\{1,\dots,N\}; \tag{21}$$

Where the variable $\overline{\boldsymbol{W}}$ is a semidefinite matrix estimated according to the procedure indicated in [19] as:

$$\overline{\boldsymbol{W}}(k)=\widetilde{\boldsymbol{A}_\iota}(k)\cdot\boldsymbol{\mu}_{\widetilde{x}}(k)\cdot\sigma_{\xi_1}^2(k)\cdot\boldsymbol{\mu}_{\widetilde{x}}(k)^T\cdot\widetilde{\boldsymbol{A}_\iota}(k)^T+\widetilde{\boldsymbol{A}_\iota}(k)\cdot\boldsymbol{\mu}_{\widetilde{x}}(k)\cdot\rho_{\xi_1(k),\xi_2(k)}\cdot\sigma_{\xi_1}(k)\cdot\sigma_{\xi_2}(k)\cdot\widetilde{\boldsymbol{D}}(k)^T+\widetilde{\boldsymbol{D}}(k)\cdot\rho_{\xi_1(k),\xi_2(k)}\cdot\sigma_{\xi_1}(k)\cdot\sigma_{\xi_2}(k)\cdot\boldsymbol{\mu}_{\widetilde{x}}(k)^T\cdot\widetilde{\boldsymbol{A}_\iota}(k)^T+\widetilde{\boldsymbol{D}}(k)\cdot\sigma_{\xi_2}^2(k)\cdot\widetilde{\boldsymbol{D}}(k)^T;\quad \forall k\in\{1,\dots,N\}; \tag{22}$$

Subsequently, dual variables $\boldsymbol{c}$ are calculated, solving the SDP optimization set in (25) for a pre-stablished value for the risk level of the individual states and inputs chance constraints.

$$\min_{\boldsymbol{M_x},\boldsymbol{M_u},\boldsymbol{B_x},\boldsymbol{B_u},\boldsymbol{V_x},\overline{\boldsymbol{W}},\boldsymbol{c},\alpha_2,\,\boldsymbol{\vartheta_x},\,\boldsymbol{\vartheta_u}} \Big(\boldsymbol{c}^T \cdot \boldsymbol{W_c} \cdot \boldsymbol{c} + \omega_{M_x} \sum_{k=1}^{N+1} \| \boldsymbol{m_{x,k}} \|_F - \omega_{B_x} \sum_{k=1}^{N+1} det\left(\sqrt[n]{\boldsymbol{b_{x,k}}}\right) - \omega_{B_u} \sum_{k=1}^{N} det\left(\sqrt[n]{\boldsymbol{b_{u,k}}}\right) + \omega_{\vartheta_x} \sum_{k=1}^{N+1} \boldsymbol{\vartheta_x} + \omega_{\vartheta_u} \sum_{k=1}^{N} \boldsymbol{\vartheta_u} + \omega_{V_x} \sum_{k=1}^{N+1} \| \boldsymbol{v_{x,k}} \|_F + \omega_{\alpha_2} \mid \alpha_2 \mid \Big) \tag{23.1}$$

$$Subject\ to:$$

$$\boldsymbol{\mu}_{\widetilde{\boldsymbol{x}}}(k+1) = (\widetilde{\boldsymbol{A_0}}(k) + \widetilde{\boldsymbol{B}}\boldsymbol{K}(k)) \cdot \boldsymbol{\mu}_{\widetilde{\boldsymbol{x}}}(k) + \widetilde{\boldsymbol{B}} \cdot \boldsymbol{c}(k); \quad \forall k \in \{1, \dots, N\}; \tag{23.2}$$

$$\boldsymbol{\mu_{\Delta u}}(k) = \boldsymbol{K}(k) \cdot \boldsymbol{\mu_{\Delta x}}(k) + \boldsymbol{c}(k); \quad \forall k \in \{1, \dots, N\}; \tag{23.3}$$

$$\boldsymbol{\mu}_{\boldsymbol{\Delta}\widetilde{\boldsymbol{x}}}(1) = 0; \tag{23.4}$$

$$\boldsymbol{m_{x,k}} = \boldsymbol{m_{x,k}}^T \succcurlyeq 0; \quad \forall k \in \{1, \dots, N+1\}; \tag{23.5}$$

$$\boldsymbol{m_{u,k}} = \boldsymbol{m_{u,k}}^T \succcurlyeq 0; \quad \forall k \in \{1, \dots, N\} \tag{23.6}$$

$$\boldsymbol{b_{x,k}} = \boldsymbol{b_{x,k}}^T \succcurlyeq 0; \quad \forall k \in \{1, \dots, N+1\}; \tag{23.7}$$

$$\boldsymbol{b_{u,k}} = \boldsymbol{b_{u,k}}^T \succcurlyeq 0; \quad \forall k \in \{1, \dots, N\}; \tag{23.8}$$

$$\boldsymbol{v_{x,k}} = \boldsymbol{v_{x,k}}^T \succcurlyeq 0; \quad \forall k \in \{1, \dots, N+1\}; \tag{23.9}$$

$$\overline{\boldsymbol{w_k}} \succcurlyeq 0; \quad \forall k \in \{1, \dots, N\}; \tag{23.10}$$

$$\begin{bmatrix} \boldsymbol{m_{x,k+1}} - \overline{\boldsymbol{w_k}} & [\widetilde{\boldsymbol{A_0}}(k) + \widetilde{\boldsymbol{B}}\boldsymbol{K}(k)]^T \\ [\widetilde{\boldsymbol{A_0}}(k) + \widetilde{\boldsymbol{B}}\boldsymbol{K}(k)] & \boldsymbol{v_{x,k}} \end{bmatrix} \succcurlyeq 0; \quad \forall k \in \{1, \dots, N\}; \tag{23.11}$$

$$\begin{bmatrix} \overline{\boldsymbol{w_k}} & [\widetilde{\boldsymbol{A_\iota}}(k)\boldsymbol{\mu}_{\widetilde{\boldsymbol{x}}}(k)) \quad \widetilde{\boldsymbol{D}}(k)] & [\widetilde{\boldsymbol{A_\iota}}(k)\boldsymbol{\mu}_{\widetilde{\boldsymbol{x}}}(k)) \quad \widetilde{\boldsymbol{D}}(k)] \\ [\widetilde{\boldsymbol{A_\iota}}(k)\boldsymbol{\mu}_{\widetilde{\boldsymbol{x}}}(k)) \quad \widetilde{\boldsymbol{D}}(k)]^T & \begin{bmatrix} \frac{1}{\sigma^2_{xi_1}(k)} & 0 \\ 0 & \frac{1}{\sigma^2_{xi_1}(k)} \end{bmatrix} & 0 \\ [\widetilde{\boldsymbol{A_\iota}}(k)\boldsymbol{\mu}_{\widetilde{\boldsymbol{x}}}(k)) \quad \widetilde{\boldsymbol{D}}(k)]^T & 0 & \begin{bmatrix} 0 & \frac{1}{cov(\xi_1(k),\xi_2(k))} \\ \frac{1}{cov(\xi_1(k),\xi_2(k))} & 0 \end{bmatrix} \end{bmatrix} \succcurlyeq \alpha_2 \ \forall k \in \{1, \dots, N\}; \tag{23.12}$$

$$\boldsymbol{m_{u,k}} = \boldsymbol{K}(k) \cdot \boldsymbol{m_{x,k}} \cdot \boldsymbol{K}(k)^T; \quad \forall k \in \{1, \dots, N\}; \tag{23.13}$$

$$\begin{bmatrix} \boldsymbol{v_{x,k}} & \boldsymbol{I} \\ \boldsymbol{I} & \boldsymbol{m_{x,k}} \end{bmatrix} \succcurlyeq 0; \quad \forall k \in \{1, \dots, N+1\}; \tag{23.14}$$

$$\begin{bmatrix} \boldsymbol{m_{x,k}} & \boldsymbol{b_{x,k}}^T \\ \boldsymbol{b_{x,k}} & \boldsymbol{I} \end{bmatrix} \succcurlyeq 0; \quad \forall k \in \{1, \dots, N+1\}; \tag{23.15}$$

$$\begin{bmatrix} \boldsymbol{m_{u,k}} & \boldsymbol{b_{u,k}}^T \\ \boldsymbol{b_{u,k}} & \boldsymbol{I} \end{bmatrix} \succcurlyeq 0; \quad \forall k \in \{1, \dots, N\}; \tag{23.16}$$

$$\sqrt{\frac{1-\epsilon_{xi}}{\epsilon_{xi}}} \cdot \left\| \begin{bmatrix} \boldsymbol{m_{x,k+1}} & \sqrt{cov(\widetilde{\boldsymbol{x}}(k), \boldsymbol{\xi_c})} \\ \sqrt{cov(\widetilde{\boldsymbol{x}}(k), \boldsymbol{\xi_c})}^T & \boldsymbol{\Sigma}^T_{\xi_c}(k)\boldsymbol{\Sigma}_{\xi_c}(k) \end{bmatrix} \widetilde{\boldsymbol{a_{x\iota}}}^T \right\| + \widetilde{\boldsymbol{a_{x\iota}}} \begin{bmatrix} \boldsymbol{\mu}_{\widetilde{\boldsymbol{x}}}(k) \\ \boldsymbol{\mu}_{\xi_c}(k) \end{bmatrix} \leq -b_{xi}(k) + \vartheta_{xi}(k); \forall i \in \{1, \dots, N_x\}; \quad \forall k \in \{1, \dots, N\}; \tag{23.17}$$

$$\sqrt{\frac{1-\epsilon_{ui}}{\epsilon_{ui}}} \cdot \left\| \begin{bmatrix} \boldsymbol{m_{u,k}} & \sqrt{cov(\Delta\boldsymbol{u}(k), \boldsymbol{\xi_c})} \\ \sqrt{cov(\Delta\boldsymbol{u}(k), \boldsymbol{\xi_c})}^T & \boldsymbol{\Sigma}^T_{\xi_c}(k)\boldsymbol{\Sigma}_{\xi_c}(k) \end{bmatrix} \boldsymbol{a_{ui}}^T \right\| + \boldsymbol{a_{ui}} \begin{bmatrix} \boldsymbol{\mu_{\Delta u}}(k) \\ \boldsymbol{\mu}_{\xi_c}(k) \end{bmatrix} \leq -b_{ui}(k) + \vartheta_{ui}(k); \forall i \in \{1, \dots, N_u\}; \quad \forall k \in \{1, \dots, N\}; \tag{23.18}$$

where the decision variables of the optimization are: $\boldsymbol{M_x} \in \mathbb{R}^{N_{\tilde{x}} \times N_{\tilde{x}} \times N+1}$ which is a three dimensional array that contains in their 2-dimensional subsets $\boldsymbol{m_{x,k}}$ the second moment of the states $\widetilde{\boldsymbol{x}}$, i.e. $\boldsymbol{m_{x,k}} = \boldsymbol{\Sigma}_{\widetilde{\boldsymbol{x}}}(k)$, being $N_{\tilde{x}} = 2 \cdot N_x$ the number of augmented states $\widetilde{\boldsymbol{x}}$, and N_x the number of states in Equation (6).

$\boldsymbol{M_u} \in \mathbb{R}^{N_u \times N_u \times N}$, which is a three dimensional array that contains in their 2-dimensional subsets $\boldsymbol{m_{u,k}}$ the second moment of the inputs $\Delta\boldsymbol{u}$, i.e. $\boldsymbol{m_{u,k}} = \boldsymbol{\Sigma_{\Delta u}}(k)$, being N_u the number of inputs in Equation (6); $\boldsymbol{B_x} \in \mathbb{R}^{N_{\tilde{x}} \times N_{\tilde{x}} \times N+1}$, is a three dimensional array that contains in their 2-dimensional subsets $\boldsymbol{b_{x,k}}$ the square root of the second moment of the system states, i.e. $\boldsymbol{b}_{x,k} = \boldsymbol{\Sigma}_{\tilde{x}}{}^{1/2}(k)$; $\boldsymbol{B}_{u} \in \mathbb{R}^{N_u \times N_u \times N}$, is a three dimensional array that contains in their 2-dimensional subsets $\boldsymbol{b}_{u,k}$ the square root of the second moment of the system inputs, i.e. $\boldsymbol{b}_{u,k} = \boldsymbol{\Sigma}_{\Delta u}{}^{1/2}(k)$; $\boldsymbol{V}_{x} \in \mathbb{R}^{N_{\tilde{x}} \times N_{\tilde{x}} \times N+1}$, is a three dimensional array that contains in their 2-dimensional subsets $\boldsymbol{v}_{x,k}$ the inverse of the second moment of the system states, i.e. $\boldsymbol{v}_{x,k} = \boldsymbol{\Sigma}_{\tilde{x}}{}^{-1}(k)$; $\boldsymbol{c} \in \mathbb{R}^{N_u \cdot N}$ is a matrix containing the dual variables $\boldsymbol{c}(k)$; $\overline{\boldsymbol{W}} \in \mathbb{R}^{N_{\tilde{x}} \times N_{\tilde{x}} \times N}$ is a three dimensional array that contains in their 2-dimensional subsets the elements $\overline{\boldsymbol{w}_k}$; $\alpha_2 \in \mathbb{R}$ is a scalar value corresponding to the minimum eigenvalues of the matrices $\overline{\boldsymbol{w}_k}$; and $\boldsymbol{\vartheta}_x \in \mathbb{R}^{N_{\tilde{x}} \cdot N}$, $\boldsymbol{\vartheta}_u \in \mathbb{R}^{N_u \cdot N}$ are vectors containing slack variables that allow the feasibility of the optimization problem, relaxing the chanced constraints.

On the other hand, the scalar ω_{M_x} is a weight factor that penalizes the Frobenius norm of the state variance matrices $\boldsymbol{m}_{x,k}$ (since the proposed strategy minimizes the states variance as much as possible); ω_{B_x} and ω_{B_u} are scalars that incentives the product of eigenvalues of matrices $\boldsymbol{b}_{x,k}$, $\boldsymbol{b}_{u,k}$ (since the calculated matrices $\boldsymbol{b}_{x,k}$ and $\boldsymbol{b}_{u,k}$ have the largest possible eigenvalues); ω_{V_x} is a weight factor that penalizes the Frobenius norm of the inverse state variance matrices $\boldsymbol{v}_{x,k}$ (since the proposed strategy minimizes also the inverse of states variance matrix); ω_{α_2} is a weight factor that penalizes the absolute value of the scalar α_2; ω_{ϑ_x} and ω_{ϑ_u} are scalars that penalizes the slack variables ϑ_x and ϑ_u, which are set to make feasible the control problem under conditions that do not accomplish with the constraints.

5. Results

The proposed hierarchical control technique has been applied to the car-sharing system study case set in [25], which is proposed for the Medellín city (Colombia). Specifically, the considered system is a 6 stations carsharing with a fleet of 854 Nissan Leaf EVs with a maximum individual energy capacity of 17.6 kWh, a minimum allowed energy content of 20%, and a maximum individual charging/discharging power of 2.2 kW.

Moreover, in the Chapter 5.1 of [25], the stochastic characterization of the variables in Equation (14), it is carried out. And here, this characterization is taken to evaluate the joint chanced constraints given in equations (25.17), (25.18), with risk levels of 15% and 30%. The results obtained of these evaluations are shown in Fig. 2-4 and summarized in Table 1, where the obtained solutions are compared regarding the results presented in the previous work [25], where the stochastic behavior of the uncertainties was omitted. Furthermore, the results presented in Table 1 include an evaluation of the control performance for the carsharing system under a perturbed condition, in which the energy consumption of EVs during travels and the number of vehicles traveling among stations deviate from their mean condition. Those variables take a value of the mean plus 2 standard deviations.

On the other hand, Fig. 2-4 depict the obtained results considering the risk levels of 15% and 30 %.

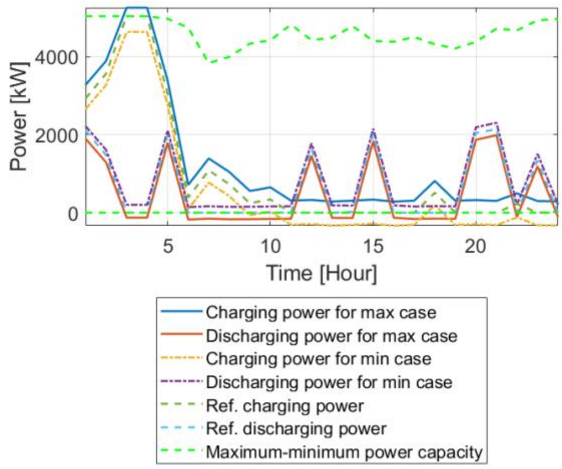


Fig 2. Calculated control actions, considering a risk level of 15%.

Where the max case indicates that the uncertainties take the mean value plus the standard deviation multiplied by the defined confidence level, and the min case indicates that the uncertainties take the mean value minus the standard deviation multiplied by the defined confidence level.

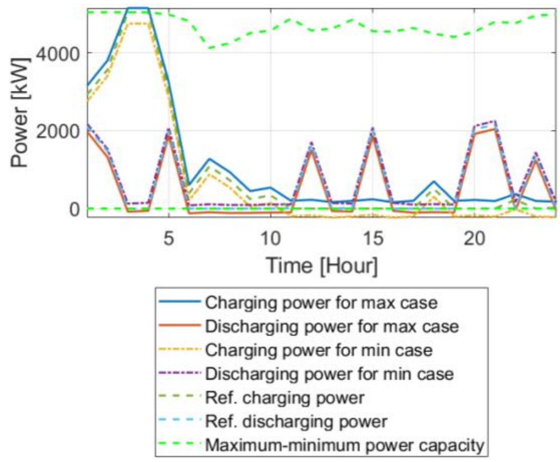
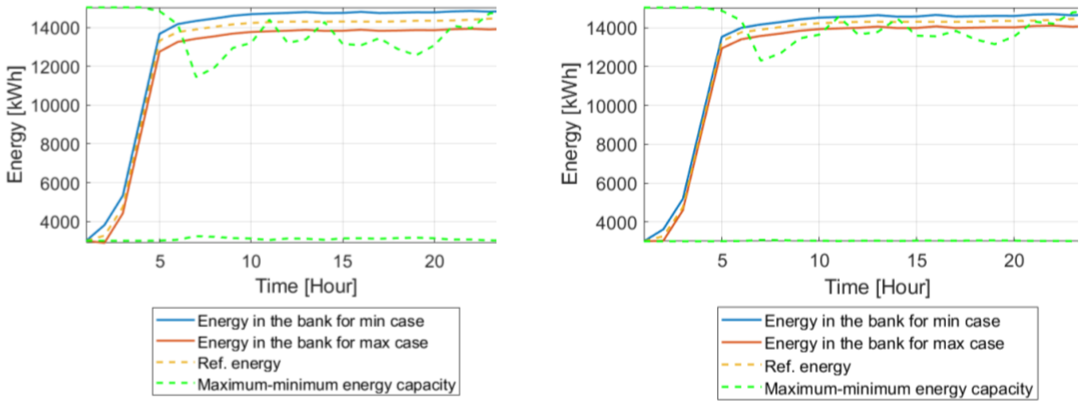


Fig 3. Calculated control actions considering a risk level of 30%.

In Fig. 2-3 it can be appreciated that the charging/discharging powers cannot be kept always inside their limits, since the control law is not able to compensate the high value of the uncertainties in the scenarios evaluated. However, the obtained control actions deviate the minimum possible from the established reference values. Furthermore, it can be noted that the charging power is the highest for the max case and the discharging power is the highest for the min case; this happens because the system requires more charging power to supply the energy consumption in travels for the max case, and, as the min case represent the minimum energy consumption in travels, the system will be available to sell more energy to the network since exists surplus energy in the batteries after travels.

Regarding the changes introducing by modifying the risk level, it can be appreciated that the deviation from reference charging/discharging values is higher in the Fig. 2, since the risk level is lower than the evaluated in Fig. 3, and the system should be able to manage higher uncertainties. This phenomenon can be also appreciated in Fig. 4.a and 4.b, where the gap between the energy content in the bank for the max case and the min case is wider for the lowest risk level evaluated (Fig. 4.a).

Additionally, upon examining Fig. 4, it can be appreciated that the energy content of vehicles stationed at the charging stations surpasses the maximum permissible energy content. This observation indicates that the system, with the given controller tuning parameters, fails to fully mitigate disturbances at the established confidence levels. Nevertheless, the inclusion of slack variables ϑ_x and ϑ_u in Equation (25) offers a viable solution that minimizes deviations from nominal conditions. Consequently, the incorporation of these slack variables enables the control problem to remain feasible even under substantial perturbations, thereby addressing the system's robustness requirements.



a) b)

Fig 4. Aggregated energy content in the battery bank, considering a risk level of a) 15% and b) 30%.

Finally, Table 1 presents a comparison for the decision-making system performance regarding the solution assuming that the uncertainties behave as their mean values, which was presented in the previous work [25]**.**

Table 1. Comparison of results regarding the nominal MPC.

-	**Risk-aware 15% of risk level**	**Risk-aware 30% of risk level**	**Nominal (solution given in [25])**
Aggregator incomes	25.748 [MCOP]	25.966 [MCOP]	26.314 [MCOP]
Constraints violation	4,1485	4,469	10,178
Execution time	15.821 [s]	16.195 [s]	41.516 [s]

From Table 1 it can be observed that proposed second level risk-aware stochastic MPC improves the decision making performance; because the obtained average value of constraints violation (calculated in [23]) is lower for the both risk levels considered than in the nominal solution, which implies that the proposed controllers improve the ability of the decision making system to reject the disturbances.

Also, it can be noted that the incomes of the aggregator consider the nominal case (first level solution with no uncertainties), are higher than the incomes for the risk-aware control cases; this agrees with the results illustrated in Fig. 2-4, since, under the evaluated conditions, the aggregator must charge the vehicles more than in the nominal or reference value to compensate the uncertainties, reducing with this, its incomes.

On the other hand, in Table 1 the execution times were also reported; from which it can be concluded that the risk-aware controller is 2.5 times faster than the first level nominal control; this implies that, for real-time applications (the requires execution time will depend of the dynamic system change rate), the proposed hierarchical controller is more suitable than the direct approach-based control schemes, such as the EMPC, since they will require higher computational burden, higher execution times, and can present stability issues as they must solve the non-linear economic optimization (see (1)) at each time step and must consider also the uncertainties, which add computational complexity to the problem; furthermore it is highlighted that the stability of the second-level tracking stochastic MPC can be proved (it was already proven in [23]), but the stability proofs of non-linear EMPCs is an open issue in the current literature. However, there is no guarantee that the solution given by the secondary control level is the global optimum of the problem, but it is the solution that can manage the system uncertainties with the less deviation possible from the initial plan, and that also considers the economy of the system.

6. Conclusion

This research paper presented a hierarchical control scheme designed for the decision-making system of a shared Electric Vehicles (EVs) aggregator actively participating in energy sales and frequency regulation reserve provision to the power network. The proposed scheme comprises two levels: the first level focuses on determining optimal economic decisions based on average uncertainty behavior, while the second level incorporates a risk-aware reference tracking controller that minimizes decision deviations from reference values. The stochastic nature of uncertainties is accounted for by considering a predefined confidence level for their characterization. The experimental results demonstrated a better performance of the proposed scheme compared to a nominal Model Predictive Control (MPC) solution that disregards the stochastic behavior of uncertainties. Specifically, the proposed scheme exhibits enhanced disturbance rejection capabilities, thereby enhancing the system's overall performance.

The proposed scheme offers advantages over existing counterparts in the literature, including EMPCs (Economic Model Predictive Controllers) and robust controllers. These advantages encompass reduced computation time, solution stability, scalability, and the ability to directly incorporate probability density functions of uncertainties and their associated confidence levels within the controller design. Moreover, this work represents an extension of the current state-of-the-art stochastic MPC approaches. Unlike previous approaches that primarily focus on linear time-invariant systems with constant constraints and zero-mean disturbances, the present study considers a linear time-varying system with time-varying constraints and non-zero mean disturbances with non-unitary variances. This extension broadens the applicability of the proposed scheme within the existing literature.

Moreover, it is suggested, as future work, the implementation of direct EMPC control schemes for the proposed model, and the use of other stochastic control techniques as the PMP-based and dynamic programming.

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References

[1] E. Sortomme and M. A. El-Sharkawi, “Optimal scheduling of vehicle-to-grid energy and ancillary services,” *IEEE Trans. Smart Grid*, vol. 3, no. 1, pp. 351–359, 2012.

[2] J. V. Kadam and W. Marquardt, “Integration of Economical Optimization and Control for Intentionally Transient Process Operation,” in *Findeisen R., Allgöwer F., Biegler L.T. (eds) Assessment and Future Directions of Nonlinear Model Predictive Control. Lecture Notes in Control and Information Sciences*, Berlin, Heidelberg: vol 358. Springer, 2007, pp. 419–434.

[3] E. Sortomme and M. A. El-Sharkawi, “Optimal combined bidding of vehicle-to-grid ancillary services,” *IEEE Trans. Smart Grid*, vol. 3, no. 1, pp. 70–79, 2012.

[4] H. Turker and M. Ponzio, “New Predictive Control Method for Optimal Minimization of Plug-in Electric Vehicle (PEV) Charging Cost with Vehicle-to-Home (V2H) capability,” *11th IEEE Int. Conf. SMART GRID - ICRERA*, 2023.

[5] H. Turker, A. Radu, S. Bacha, D. Frey, J. Richer, and P. Lebrusq, “Optimal charge control of electric vehicles in parking stations for cost minimization in V2G concept,” *3rd Int. Conf. Renew. Energy Res. Appl. ICRERA 2014*, pp. 945–951, 2014.

[6] E. A. Buehler, J. A. Paulson, and A. Mesbah, “Lyapunov-based stochastic nonlinear model predictive control: Shaping the state probability distribution functions,” *Proc. Am. Control Conf.*, vol. 2016-July, no. Section IV, pp. 5389–5394, 2016.

[7] S. Liu and J. Liu, “Economic Model Predictive Control with Zone Tracking,” *IFAC-PapersOnLine*, vol. 51, no. 20, pp. 16–21, 2018.

[8] M. Gonzalez Vaya and G. Andersson, “Optimal Bidding Strategy of a Plug-In Electric Vehicle Aggregator in Day-Ahead Electricity Markets under Uncertainty,” *IEEE Trans. Power Syst.*, vol. 30, no. 5, pp. 2375–2385, 2015.

[9] H. Liu, Z. Hu, Y. Song, J. Wang, and X. Xie, “Vehicle-to-Grid Control for Supplementary Frequency Regulation Considering Charging Demands,” *IEEE Trans. Power Syst.*, vol. 30, no. 6, pp. 3110–3119, 2015.

[10] S. R.- Álvarez, J. Patiño, A. Márquez, and J. Espinosa, “Optimal Design for an Electrical Hybrid Micro Grid in Colombia Under Fuel Price Variation,” *Int. J. Renew. Energy Res.*, vol. 7, no. 4, 2017.

[11] S. M. Sotoudeh and B. Homchaudhuri, “A Robust MPC-based Hierarchical Control Strategy for Energy Management of Hybrid Electric Vehicles in Presence of Uncertainty,” *Proc. Am. Control Conf.*, vol. 2020-July, pp. 3065–3070, 2020.

[12] C. Hou, M. Ouyang, L. Xu, and H. Wang, “Approximate Pontryagin’s minimum principle applied to the energy management of plug-in hybrid electric vehicles,” *Appl. Energy*, vol. 115, pp. 174–189, 2014.

[13] J. Löfberg, “Minimax approaches to Robust Model Predictive Control,” Linköping University, 2014.

[14] B. Kouvaritakis and M. Cannon, “Stochastic Model Predictive Control,” *Encycl. Syst. Control*, pp. 1–9, 2014.

[15] S. Abderrahim, M. Allouche, and M. Chaabane, “A New Robust Control Strategy for a Wind Energy Conversion System Based on a T-S Fuzzy Model,” *Int. J. Smart grid*, vol. 4, no. 2, 2020.

[16] S. Zymler, D. Kuhn, and B. Rustem, “Distributionally robust joint chance constraints with second-order moment information,” *Math. Program.*, vol. 137, no. 1–2, pp. 167–198, 2013.

[17] R. Bellman, “Dynamic programming,” *Science (80-.).*, vol. 153, no. 3731, pp. 34–37, 1966.

[18] C. Hou, L. Xu, H. Wang, M. Ouyang, and H. Peng, “Energy management of plug-in hybrid electric vehicles with unknown trip length,” *J. Franklin Inst.*, vol. 352, no. 2, pp. 500–518, 2015.

[19] M. Farina and R. Scattolini, “Model predictive control of linear systems with multiplicative unbounded uncertainty and chance constraints,” *Automatica*, vol. 70, pp. 258–265, 2016.

[20] C. Goebel and H. A. Jacobsen, “Aggregator-Controlled EV Charging in Pay-as-Bid Reserve Markets with Strict Delivery Constraints,” *IEEE Trans. Power Syst.*, vol. 31, no. 6, pp. 4447–4461, 2016.

[21] M. Akil, E. Dokur, and R. Bayindir, “A coordinated EV Charging Scheduling Containing PV System,” *Int. J. Smart Grid*, vol. 6, no. 3, pp. 65–71, 2022.

[22] E. Kilic, M. Akil, R. Bayindir, A. Sebati, and R. Malek, “Electric Vehicles Charging Management with Monte Carlo Simulation,” *2021 10th International Conference on Renewable Energy Research and Application (ICRERA)*, 2021. [Online]. Available:

https://ieeexplore.ieee.org/document/9598807. [Accessed: 15-Jul-2023].

[23] S. Ruiz-Alvarez, "Methodology to design an optimal decision making system for the operator of a shared electric vehicle fleet providing electrical ancillary services," Universidad Nacional de Colombia, 2021.

[24] B. Kouvaritakis and M. Cannon, *Model Predictive Control Classical, Robust and Stochastic*. Advanced Textbooks in Control and Signal Processing Series- Springer, 2008.

[25] S. Ruiz-Álvarez and D. Gómez-Ramírez, "Optimal Management Strategy for a Shared EVs Aggregator Participating in Electricity and Frequency Regulation Reserves Markets," *Technol. Econ. Smart Grids Sustain. Energy*, vol. 7, 2021.

[26] J. A. Paulson, E. A. Buehler, R. D. Braatz, and A. Mesbah, "Stochastic model predictive control with joint chance constraints," *Int. J. Control*, vol. 93, no. 1, pp. 126–139, 2020.