Solar Radiation Models for the City of Tirana, Albania

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Received: 31.03.2014 Accepted: 01.05.2014

Abstract- Mathematical models for evaluating the solar radiation potential are not available for every country or specific sites. In this paper, different solar radiation models (linear, exponential, power) for the city of Tirana-Albania, are built and tested. These models are used to estimate the monthly average total solar radiation on horizontal surface, based on measured data. Measured data include solar radiation on horizontal surface and sunshine duration data, which are used for the development of the models. Calculated and measured values are compared and evaluated by using statistical test methods. Calculated values obtained from the proposed solar radiation models show a good agreement with the measurements. In the best solar radiation model, the percentage error is $e_{\text{max}} = 3.452\%$, MBE $= -0.001465$ kWh/(m2·day), MPE $= -0.015\%$, RMSE $= 0.074$ kWh/(m2·day), and $R^2 = 0.998$.

Keywords- total solar radiation; horizontal surface; sunshine duration; statistical test values; solar radiation models.

1. Introduction

Because of its geographical location, Albania belongs to the Mediterranean climate belt with hot and dry summers and mild winters. The average annual precipitation over the country is about 1485 mm [1]. Albania is considered as a country with an upper-average radiation potential, which varies between 1185 and 1700 kWh/m2 year. The average annual number of sunny days is $240 - 260$ days/year [2]. Tirana is the capital and is situated in the central part of the country. Geographical coordinates at 110 m above the sea level, are latitude $41.33^\circ$N and longitude $19.82^\circ$E [3]. For Tirana, the average annual sunshine duration is about 2500 h/year and the average annual total solar radiation is 1500 kWh/(m2·year) [2].

Solar radiation reaching the top layers of atmosphere is well defined and can be calculated easily. Meanwhile, the solar irradiance reaching the ground depends mainly on location and atmospheric conditions.

For Albania, the currently available solar radiation data are valueless for an appropriate evaluation of solar-based conversion systems. To obtain such data, a network of meteorological stations equipped with solar radiation sensors, is required. It must be emphasized that investment and maintenance costs present a barrier for Albania.

The main purpose of this work is to build a monthly medium-term forecasting model based on meteorological data for the city of Tirana-Albania for the estimation of total solar radiation on a horizontal surface. These data include solar radiation on a horizontal surface and sunshine duration.

The proposed forecasting models for estimating total solar radiation on horizontal surface are evaluated after the comparison of their statistical parameters. The statistical parameters used in this work, include the percentage error ($e$), the mean bias error (MBE), the mean percentage error (MPE), the root mean square error (RMSE), and the coefficient of determination ($R^2$). Finally, measured values are compared to calculated ones and evaluated by the statistical methods mentioned above.

2. Materials and methods

This paper presents an approach to build and evaluate different solar radiation models. The models are used to estimate the monthly average daily total solar radiation on a horizontal surface, based on measured data. Measured data include monthly-average daily total solar radiation on a horizontal surface and monthly-average daily sunshine duration, which are used for the development of the models.
2.1. Input Parameters

Solar radiation data measurements constitute the best source of information for estimating the average incident solar radiation. Sunshine duration data are widely available from many stations in many countries and are usually based on data obtained by the Campbell-Stokes instruments [4]. For the case of Tirana, monthly average daily total solar radiation data on horizontal surface ($\bar{H}$) are used [2]. Data of sunshine duration ($\bar{n}$) are provided by Wetter [5]. Recommended average days for each month ($n$) are given by Klein [6] and here they refer to a whole year. These values are shown in Table 1, where input parameters are also included.

### Table 1. Input parameters to the solar radiation models

<table>
<thead>
<tr>
<th>Month</th>
<th>$\bar{H}$ (kWh/m²·day)</th>
<th>$\bar{n}$ (h/day)</th>
<th>$n$ (-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>1.830</td>
<td>4.1</td>
<td>17</td>
</tr>
<tr>
<td>February</td>
<td>2.468</td>
<td>4.4</td>
<td>47</td>
</tr>
<tr>
<td>March</td>
<td>3.346</td>
<td>5.1</td>
<td>75</td>
</tr>
<tr>
<td>April</td>
<td>4.468</td>
<td>6.8</td>
<td>105</td>
</tr>
<tr>
<td>May</td>
<td>5.602</td>
<td>8.6</td>
<td>135</td>
</tr>
<tr>
<td>June</td>
<td>6.477</td>
<td>9.9</td>
<td>162</td>
</tr>
<tr>
<td>July</td>
<td>6.781</td>
<td>11.4</td>
<td>198</td>
</tr>
<tr>
<td>August</td>
<td>5.990</td>
<td>10.6</td>
<td>228</td>
</tr>
<tr>
<td>September</td>
<td>4.631</td>
<td>8.8</td>
<td>258</td>
</tr>
<tr>
<td>October</td>
<td>3.190</td>
<td>7</td>
<td>28</td>
</tr>
<tr>
<td>November</td>
<td>1.981</td>
<td>4.2</td>
<td>318</td>
</tr>
<tr>
<td>December</td>
<td>1.546</td>
<td>2.8</td>
<td>344</td>
</tr>
</tbody>
</table>

The procedure for the calculation of necessary parameters used in the suggested solar radiation models is presented below.

The solar constant used in this paper is the value as defined in [Duffie and Beckman [4], Thekaekara [7], ASTM [8], and NASA [9]]:

$$I_{sc} = 1353 \text{ W/m}^2 = 1.353 \text{ kW/m}^2$$

The monthly average daily extraterrestrial solar radiation on a horizontal surface, for latitudes from ($-60^\circ$ to $60^\circ$) can be calculated by using the below equation [10]:

$$\bar{H}_o = \frac{24}{\pi} \cdot I_{sc} \cdot r \cdot \left[ \cos(\phi) \cdot \cos(\delta) \cdot \sin(\omega_s) + \frac{\pi}{180} \cdot \omega_s \cdot \sin(\phi) \cdot \sin(\delta) \right]$$  \hspace{1cm} (1)

The eccentricity correction factor of the Earth’s orbit can be computed as below [10]:

$$r = 1 + 0.033 \cdot \cos\left(\frac{360n}{365}\right)$$ \hspace{1cm} (2)

The declination angle is given as [11]:

$$\delta = 23.45 \cdot \sin\left[\frac{360 \cdot \left(\frac{284+n}{365}\right)}{2}\right]$$ \hspace{1cm} (3)

The sunset hour angle ($\omega_s$), for the zenith angle to be $90^\circ$ is given as follows [4]:

$$\cos(\omega_s) = -\tan(\phi) \cdot \tan(\delta)$$ \hspace{1cm} (4)

For a given month, the number of daylight hours calculated by equation (5)[4]. The monthly average daylight hours ($\bar{N}$) refers to ($n$).

$$\bar{N} = \frac{2}{15} \cdot \omega_s$$ \hspace{1cm} (5)

2.2. Models

The proposed models correlate ($\bar{H}/\bar{H}_o$) and ($\bar{n}/\bar{N}$). In this paper, several solar radiation models which offer good results are built and evaluated statistically. The aim of the present study is to introduce a specific model for Tirana, Albania.

### Linear model

This type of modeling was first introduced by Ångström [12] and later by Hoyt [13] and Liu-Jordan [14]. By reconsidering Ångström’s model, Page [15] and Prescott [16] presented their method which is based on the monthly average daily extraterrestrial solar radiation on a horizontal surface [4]. Many researchers have used this model to develop empirical correlations for specific sites.

Some researchers have found that regression coefficients in the linear model are site-dependent, i.e., they depend on latitude, climatic conditions, season, etc [15,17-26].

In our calculations, the following expression is used.

$$\frac{\bar{H}}{\bar{H}_o} = a + b \cdot \frac{\bar{n}}{\bar{N}}$$ \hspace{1cm} (6)

### Exponential model

This model is used by several researchers [27-32].

$$\frac{\bar{H}}{\bar{H}_o} = a \cdot \exp\left[b \cdot \frac{\bar{n}}{\bar{N}}\right]$$ \hspace{1cm} (7)

### Gaussian model

This model is similar to the exponential model, but it has one extra coefficient [33].

$$\frac{\bar{H}}{\bar{H}_o} = a \cdot \exp\left[-\left(\frac{b \cdot \bar{n}}{\bar{N}}\right)^2\right]$$ \hspace{1cm} (8)

### Power model 1 (1-st type)

Power models are exploited by Allen [23], Lewis [32], Richardson [34], Ododo et al. [35], Swartman et al. [36].

$$\frac{\bar{H}}{\bar{H}_o} = a \cdot \left(\frac{\bar{n}}{\bar{N}}\right)^b$$ \hspace{1cm} (9)

Power model 2 (2-nd type)

This model is similar to the power model 1, but it has one more coefficient [37].

$$\frac{\bar{H}}{\bar{H}_o} = a \cdot \left(\frac{\bar{n}}{\bar{N}}\right)^b + c$$ \hspace{1cm} (10)

### Weibull model

The Weibull distribution model is widely used in reliability and life data analysis. In this case, a two-parameter Weibull model is used [38].
\[
\frac{\bar{H}}{\bar{H}_{0}} = a \cdot b \cdot \left( \frac{n}{\bar{N}} \right)^{(b-1)} \cdot \exp \left[ -a \cdot \left( \frac{n}{\bar{N}} \right)^{b} \right] (11)
\]

It must be emphasized, that for this paper, other mathematical models were also considered, but the obtained results were much poorer.

2.3. Comparison Methods

The performance of the proposed solar radiation models was evaluated on the basis of the following statistical tests: the percentage error ($\varepsilon$), the mean bias error (MBE), the mean percentage error (MPE), the root mean square error (RMSE), and the coefficient of determination ($R^2$). These tests are applied commonly in comparing models of solar radiation estimations[39-54].

The parameter $\varepsilon$ represents the difference between the measured ($\bar{H}_{im}$) and the calculated ($\bar{H}_{ic}$) values of $\bar{H}$ for the $i$-th month, as a percent of the measured value. In this instance, an $\varepsilon$ value close to zero is preferable. Also, an $\varepsilon$ between $\pm 10\%$ is considered acceptable and defined as:

\[
\varepsilon = \left( \frac{\bar{H}_{im} - \bar{H}_{ic}}{\bar{H}_{im}} \right) \cdot 100\% (12)
\]

The MBE is an indicator for the average deviation of the predicted values from the measured data. The MBE provides information on the long-term performance of any model. A low MBE value is desirable. A positive value gives the average amount of over estimation of the calculated values. The MBE is defined as:

\[
\text{MBE} = \frac{1}{n} \cdot \sum_{i=1}^{n} (\bar{H}_{ic} - \bar{H}_{im}) (13)
\]

The MPE is an indicator of accuracy, in which it usually expresses accuracy as percentage. A low value of MPE is desirable and is defined as:

\[
\text{MPE} = \frac{1}{n} \cdot \sum_{i=1}^{n} \frac{\bar{H}_{im} - \bar{H}_{ic}}{\bar{H}_{im}} \cdot 100\% (14)
\]

The RMSE provides information on the short-term performance of the models and is a measure of the variation of the predicted values around the measured data. A low value of RMSE is desirable and is defined as:

\[
\text{RMSE} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (\bar{H}_{ic} - \bar{H}_{im})^2} (15)
\]

The $R^2$ is a measure that allows us to determine how certain a prediction provided from a model is. A value of $R^2$ close to the unit is desirable. It is defined as:

\[
R^2 = 1 - \frac{\sum_{i=1}^{n} \left( \bar{H}_{ic} - \bar{H}_{im} \right)^2}{\sum_{i=1}^{n} \left( \bar{H}_{im} - \bar{H} \right)^2} (16)
\]

3. Model development and evaluation

In Table 2 by using the equations (1-5), the monthly average daily extraterrestrial solar radiation on a horizontal surface ($\bar{H}_{0}$), the monthly average daylighthours ($\bar{N}$), and the ratios of ($\bar{H}/\bar{H}_{0}$) and ($\bar{n}/\bar{N}$) are shown.

<table>
<thead>
<tr>
<th>Month</th>
<th>$\bar{H}_{0}$ (kWh/m²·d)</th>
<th>$\bar{N}$ (h/day)</th>
<th>$\bar{H}/\bar{H}_{0}$ (%)</th>
<th>$\bar{n}/\bar{N}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>3.998</td>
<td>9.4</td>
<td>0.4578</td>
<td>0.437</td>
</tr>
<tr>
<td>February</td>
<td>5.493</td>
<td>10.4</td>
<td>0.4493</td>
<td>0.421</td>
</tr>
<tr>
<td>March</td>
<td>7.450</td>
<td>11.7</td>
<td>0.4491</td>
<td>0.435</td>
</tr>
<tr>
<td>April</td>
<td>9.516</td>
<td>13.1</td>
<td>0.4695</td>
<td>0.518</td>
</tr>
<tr>
<td>May</td>
<td>10.994</td>
<td>14.3</td>
<td>0.5095</td>
<td>0.600</td>
</tr>
<tr>
<td>June</td>
<td>11.604</td>
<td>14.9</td>
<td>0.5582</td>
<td>0.663</td>
</tr>
<tr>
<td>July</td>
<td>11.284</td>
<td>14.7</td>
<td>0.6009</td>
<td>0.778</td>
</tr>
<tr>
<td>August</td>
<td>10.075</td>
<td>13.6</td>
<td>0.5945</td>
<td>0.778</td>
</tr>
<tr>
<td>September</td>
<td>8.186</td>
<td>12.3</td>
<td>0.5657</td>
<td>0.718</td>
</tr>
<tr>
<td>October</td>
<td>6.059</td>
<td>10.9</td>
<td>0.5265</td>
<td>0.645</td>
</tr>
<tr>
<td>November</td>
<td>4.343</td>
<td>9.7</td>
<td>0.4561</td>
<td>0.435</td>
</tr>
<tr>
<td>December</td>
<td>3.593</td>
<td>9.1</td>
<td>0.4303</td>
<td>0.309</td>
</tr>
</tbody>
</table>

The regression coefficients (a, b, and c) depend on the measured values of ($\bar{H}$) and ($\bar{n}$). The coefficients for each considered solar radiation model with the best results are shown in Table 3.

Table 3: Considered models and their specific coefficients for the city of Tirana

<table>
<thead>
<tr>
<th>Model number</th>
<th>Model type</th>
<th>Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Linear polynomial</td>
<td>$a = 0.2885$, $b = 0.3867$</td>
</tr>
<tr>
<td>2</td>
<td>Exponential</td>
<td>$a = 0.3253$, $b = 0.7737$</td>
</tr>
<tr>
<td>3</td>
<td>Gaussian</td>
<td>$a = 9.445 \cdot 10^9$, $b = 61.79$, $c = 12.59$</td>
</tr>
<tr>
<td>4</td>
<td>Power model 1</td>
<td>$a = 0.6489$, $b = 0.416$</td>
</tr>
<tr>
<td>5</td>
<td>Power model 2</td>
<td>$a = 0.3536$, $b = 2.336$, $c = 0.4036$</td>
</tr>
<tr>
<td>6</td>
<td>Weibull</td>
<td>$a = 0.562$, $b = 1.764$</td>
</tr>
</tbody>
</table>

To evaluate the performance of the proposed solar radiation models through the equations (6-11), the correlation between the calculated ratios of ($\bar{H}/\bar{H}_{0}$) and ($\bar{n}/\bar{N}$) for each model, is shown in Fig. 1.
Fig. 1. Correlation between the ratios of \( \frac{\bar{H}}{H_0} \) and \( \frac{\bar{n}}{\bar{N}} \) for each proposed model

From Fig. 1, a linear tendency between the calculated values of the ratio \( \frac{\bar{H}}{H_0} \) towards the calculated values of the ratio \( \frac{\bar{n}}{\bar{N}} \) should be noticed. This way, it can be said that there exists a strong correlation between these ratios for the selected models. The correlation coefficient takes the highest value from the power model 2, which is \( R_{p2} = 0.9992 \). The lowest value of \( R \) is obtained from the Weibull model, which is \( R_{WG} = 0.9946 \). Based on the correlation coefficient values, it can be said that the formulation according to power model 2 gives the best solar radiation prediction for Tirana.

The selection of the most appropriate solar radiation model, based on the correlation coefficient values only is not sufficient. Consequently, the use of other statistical tests methods is performed.

In Table 4, statistical test values obtained from the exploitation of statistical test methods for the six considered models are shown.

It can be noticed that the minimum percentage error value varies in the range of \([-2.187\% , -7.146\%] \) for the power model 2, and the Weibull model, respectively. The maximum percentage error value varies in the range of \([3.435\%, 12.539\%] \) for the Gaussian model and the Weibull model, respectively.

The minimum value of \( MBE \) is \( MBE_{min} = -1.312 \cdot 10^{-3} \) kWh/(m²·day) for the Weibull model, while the maximum value is \( MBE_{max} = 26.185 \cdot 10^{-3} \) kWh/(m²·day) for the Gaussian model.
The minimum value of MPE is $\text{MPE}_{\text{min}} = -0.015\%$ for the power model 2, while its maximum value is $\text{MPE}_{\text{max}} = -0.609\%$ for the Gaussian model.

The minimum value of the RMSE is $\text{RMSE}_{\text{min}} = 0.074\text{ kWh}/(\text{m}^2\cdot\text{day})$ for the power models, while its maximum value is $\text{RMSE}_{\text{max}} = 0.188\text{ kWh}/(\text{m}^2\cdot\text{day})$ for the Weibull model.

<table>
<thead>
<tr>
<th>Model no.</th>
<th>Model type</th>
<th>$\epsilon_{\text{min}}$ (%)</th>
<th>$\epsilon_{\text{max}}$ (%)</th>
<th>$\bar{\text{MBE}}$ (kWh/m$^2$·day)</th>
<th>$\text{MPE}$ (%)</th>
<th>$\text{RMSE}$ (kWh/m$^2$·day)</th>
<th>$R^2$ (-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Linear model</td>
<td>-4.138</td>
<td>5.205</td>
<td>2.554 · 10^{-3}</td>
<td>-0.031</td>
<td>0.095</td>
<td>0.997</td>
</tr>
<tr>
<td>2</td>
<td>Exponential model</td>
<td>-3.468</td>
<td>4.000</td>
<td>2.833 · 10^{-3}</td>
<td>-0.023</td>
<td>0.081</td>
<td>0.998</td>
</tr>
<tr>
<td>3</td>
<td>Gaussian model</td>
<td>-4.093</td>
<td>3.435</td>
<td>26.185 · 10^{-3}</td>
<td>-0.609</td>
<td>0.084</td>
<td>0.998</td>
</tr>
<tr>
<td>4</td>
<td>Power model 1</td>
<td>-5.101</td>
<td>7.554</td>
<td>3.295 · 10^{-3}</td>
<td>-0.028</td>
<td>0.074</td>
<td>0.996</td>
</tr>
<tr>
<td>5</td>
<td>Power model 2</td>
<td>-2.187</td>
<td>3.452</td>
<td>-1.465 · 10^{-3}</td>
<td>-0.015</td>
<td>0.074</td>
<td>0.998</td>
</tr>
<tr>
<td>6</td>
<td>Weibull model</td>
<td>-7.146</td>
<td>12.539</td>
<td>-1.312 · 10^{-3}</td>
<td>0.155</td>
<td>0.188</td>
<td>0.989</td>
</tr>
</tbody>
</table>

The measured values ($\bar{H}_{im}$) and the calculated values ($\bar{H}$) for each solar radiation model are shown in Fig. 2. It can be noticed that the power model 1, power model 2, linear, exponential, and Gaussian models yield the best results in approximating the measured values ($\bar{H}$). This can also be noticed, from the percentage error which is within the limit of $\pm 10\%$. The Weibull model is the weakest considered model with errors, which vary between $\epsilon_{\text{min}} = -7.146\%$ to $\epsilon_{\text{max}} = 12.539\%$. The highest percentage error occurs in December and it diminishes in the other months. The value of the maximum percentage error is about the limit of $\pm 10\%$.

Other mathematical models (higher degree polynomial, rational, Fourier, etc.) were also tested and the results were much more poorer. Therefore, the results from those models are not presented in this study.

Fig. 2. Measured and calculated values of $\bar{H}$ for each solar radiation model

4. Conclusions

In this paper, six different solar radiation models were considered and evaluated thanks to the measured values of $\bar{H}$ for the city of Tirana, Albania. The evaluation of these models was performed through the exploitation of the main statistical test methods. The minimum value in percentage error, is achieved in the power model 2 as $\epsilon_{\text{min}} = -2.187\%$, in $\text{MBE}$ is $\text{MBE}_{\text{min}} = -1.312 \times 10^{-3}$ kWh/(m$^2$·day) in the Weibull model, in $\text{MPE}$ is $\text{MPE}_{\text{min}} = -0.015\%$ in the power model 2, in $\text{RMSE}$ is $\text{RMSE}_{\text{min}} = 0.074$ kWh/(m$^2$·day) in the power model 1 and 2.

The maximum value for the coefficient of determination is $R^2_{\text{max}} = 0.998$ in the exponential, the Gaussian, and the power model 2.

As a conclusion, based on the calculated statistical test values, the power model 2 represents the most appropriate solar radiation model for the city of Tirana, Albania.

Nomenclature

- $a$, $b$, $c$ [-]: regression coefficients
- $\cos$: cosine
- $\exp$: exponential
- $\bar{H}$ [kWh/(m$^2$·day)]: monthly average daily total solar radiation on horizontal surfaces
- $\bar{H}_{im}$ [kWh/(m$^2$·day)]: measured values of $\bar{H}$ for the $i$-th month
- $\bar{H}_{ic}$ [kWh/(m$^2$·day)]: calculated values of $\bar{H}$ for the $i$-th month
- $\bar{H}_{ho}$ [kWh/(m$^2$·day)]: monthly average daily extraterrestrial solar radiation on a horizontal surface
- $I_{sc}$ [kW/m$^2$]: solar constant
- $\text{MBE}$ [kWh/(m$^2$·day)]: mean bias error
- $\text{MPE}$ [%]: mean percentage error
- $n$ [-]: day number, total number of observations
- $\bar{n}$ [h/day]: monthly average daily hours of bright sunshine
- $\bar{N}$ [h/day]: monthly average number of daylight hours
- $r$ [-]: eccentricity correction factor of the Earth’s orbit
- $R$ [-]: correlation coefficient
INTERNATIONAL JOURNAL OF RENEWABLE ENERGY RESEARCH
Altin Maraj et al., Vol.4, No.2, 2014

R² [-]: coefficient of determination
RMSE[kWh/(m² day)]: root mean square error
sin[-]: sine
max: maximum;
min: minimum
δ [°]: declination angle
e [%]: percentage error
θz [°]: zenith angle
π [-]: constant
ϕ [°]: latitude
ωs [°]: sunset hour angle

Conflict of interests

The authors declare that there is no conflict of interests regarding the publication of this article.

References


