Optimal Expansion Planning of Distribution System Capacity with Respect to Distributed Generations

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Abstract- Nowadays, distributed generation (DG) is a new option in the power systems to meet the electrical demand growth as well as conventional methods such as substation reinforcement and feeder replacement. This paper presents a new approach to solve single and multi-objective distribution system expansion planning problem including DG and conventional method simultaneously. Since this optimization problem has a nonlinear complex nature, classical mathematical method cannot guarantee to achieve the global optimum solution. So, to overcome this encumbrance, a heuristic evolutionary algorithm based on binary particle swarm optimization (PSO) is proposed. The objective functions of this optimization problem are total expansion cost, voltage deviation, and system losses. The model resolves decision variables as follows: location and size of the DG units, new transformers, and upgraded feeders. This paper proposes several non-dominated solutions, so the decision maker can choose the optimal solution based on the importance of the different objectives. The 9-bus and 30-bus distribution test systems are utilized to show the effectiveness of the proposed method.

Keywords Planning, Distribution System, Distributed Generation, Optimization, Binary Particle Swarm Optimization, Weighted Aggregation Method.

1. Introduction

Distribution systems need flexible and intelligent planning methodologies in order to determine the best size, location, and type of facilities which should be installed in the network to meet demand for future times [1,2]. To achieve this main goal, there are several planning motivations such as cost reduction, voltage profile improvement, and active power loss reduction for planners to focus on an optimal planning [3]. In this regard, DG is an attractive option which can satisfy these objectives and distribution system technical constraints [4]. Thus, the conventional distribution system planning (DSP) which considers substation reinforcement and feeders upgrading can be integrated with distribution generation optimal placement problem to establish an advanced DSP problem. This problem will be a nonlinear, non-convex, non-differentiable, and constrained optimization problem which can be demonstrated by binary decision variables.

Traditionally, mathematical optimization tools, such as nonlinear programming (NLP) [5,6], Benders’ decomposition [7], and dynamic programming [8] have been used to solve optimization problem for small scale systems. The main difficulties in solving this problem with these tools are related to the combinatorial nature of the problem with numerous local optimal solutions, especially in large scale systems [9]. So, some population-based meta-heuristic algorithms have been presented for solving this problem [10], e.g. genetic algorithm (GA) [11], and particle swarm optimization [12].

Recently, the DSP problem has received much attention in the literature. In [13], a method for peak load cutting based on GA is proposed. This model determines the optimal planning scheme including the network feeders, location and sizing of DG to minimize total cost as an objective function. Conventional method without DG is discussed in [14]. In this model nonlinear objective function has approximated with linear one. Unlike the previous, another method in [15] has considered DG without conventional method in single and multi-objective functions. Some papers consider this problem as a dynamic and probabilistic problem with respect to the timing of the investment decisions and uncertainties nature.
of loads, resources, etc. [16,17]. There are some other surveys which note objectives from viewpoint of distribution generator owner [18].

This paper tries to fill the gap of non-simultaneous consideration of conventional planning and DG as an alternative option in a single and multi-objective non-linear and non-convex model without any approximation. Also in this work, by proposing non-dominated solutions, the decision maker can select the best solution based on the importance of the different objectives. The proposed method in this paper is a static model and assumes that all investments are done in the beginning of the planning horizon. The decisions regarding new equipment type, location, and capacity are determined by distribution system owner.

The paper is organized as follows: Problem formulation is presented in Section 2. Optimization method and solution algorithm is explained in section 3. The implementation of the proposed algorithm on the DSP problem and results are shown in the section 4. This paper is concluded in the final section.

2. Problem Description, Modelling and Formulation

2.1. Objective Functions

In this section, the mathematical formulation of the planning problem is presented.

2.1.1. Total Expansion Cost (TEC)

The cost function is consisted of the cost of substation reinforcement, DGs, upgrading feeders, and the active power losses. The fixed cost or investment cost is done at the beginning of the planning horizon. The variable cost mainly depends on the loading of the equipment during the operation period. Total cost function is formulated as follows:

\[ J_{\text{TEC}} = C_U + C_{DG} + C_F + C_L \]  

(1)

Where

- \( J_{\text{TEC}} \) Objective function of the total expansion cost
- \( C_U \) Fixed and variable cost for substation expansion
- \( C_{DG} \) Fixed and variable cost for DG
- \( C_F \) Fixed cost of upgrading the feeders
- \( C_L \) Variable cost for total system losses

Each part of cost functions is expressed as below:

\[ C_U = \sum_{m=1}^{n} \sum_{l=1}^{M_l} C_{U,m,l} \cdot \sigma_{t,l} + 8760 \cdot C_{EM} \cdot \sum_{i=1}^{N} \left( \beta_i \cdot \sum_{j=1}^{N_{DG,i}} \left( C_{DG,j} \cdot S_{DG,j} + \sigma_{DG,j} \right) \right) \]  

(2)

\[ C_{DG} = \sum_{m=1}^{n} \sum_{l=1}^{M_l} \sum_{j=1}^{N_{DG,i}} \left( C_{DG,F,j} \cdot S_{DG,j} + \sigma_{DG,j} \right) \]  

(3)

\[ C_F = \sum_{i=1}^{N} C_{FG,i} \cdot \sigma_{i} \]  

(4)

\[ C_L = 8760 \cdot C_{EM} \cdot \sum_{i=1}^{N} \beta_i \left( \sum_{j=1}^{N_{DG,i}} R_j \cdot \left| I_j \right|^2 \right) \]  

(5)

Where

- \( C_{EM} \) Electricity market price ($/MWh)
- \( C_{TR,F,i,j} \) Fixed cost of the j-th transformer in the i-th substation ($/unit)
- \( C_{DG,F,i,j} \) Fixed cost of the j-th DG unit in the i-th load node ($/unit)
- \( C_{DG,O,i,j} \) Operation cost of the j-th DG unit in the i-th load node ($/MWh)
- \( \sigma_{DG,i,j} \) Binary decision variable of the j-th DG unit in the i-th load node
- \( \sigma_{TR,i,j} \) Binary decision variable of the j-th transformer in the i-th substation
- \( \beta_i \) Binary decision variable of the l-th feeder
- \( S_{DG,i,j} \) Power generated from the j-th DG unit in the i-th load node (MVA)
- \( P_{DG,i,j} \) Total active power transferred from electricity market in the i-th substation (MW)
- \( P_{DFL,i} \) Power factor of the j-th DG unit in the i-th load node
- \( L \) Feeder’s set
- \( C_{FL,i} \) upgrade cost of the l-th feeder ($)
- \( I_j \) Resistance of the l-th feeder
- \( I_j \) Current of the l-th feeder
- \( \beta \) Present worth factor
- \( d \) Discount factor
- \( N \) Set of suitable nodes for substation expansion
- \( N_{DG} \) Maximum number of transformers at each node
- \( M \) Set of suitable nodes for installing DG units (load nodes)
- \( M_{DG} \) Maximum number of DG considered at each node
- \( BK_1 \) Back-up protection DG unit in the i-th load node
- \( l \) Incremental time intervals (in years)
- \( t \) Horizon planning year (in years)

The present worth factor, \( \beta = \frac{1}{(1+d)} \), is used to convert future costs to present values [19].

2.1.2. Total Voltage Deviation (TVD)

One of the benefits of DG in distribution is the improvement of the voltage profile of the system. Voltage profile can be improved mainly because of the real and reactive power provided by DG. The proposed \( J_{\text{TVD}} \) is defined as the difference between calculated voltage from load flow and nominal voltage in per unit. This objective minimizes the deviations in voltage magnitudes at each bus that can be expressed through the following equation [20]:

\[ J_{\text{TVD}} = \sum_{i=1}^{N} \left| V_i - V_{nom,i} \right| \]  

(6)

Where

- \( J_{\text{TVD}} \) Objective function of the total voltage deviation

The paper tries to fill the gap of non-convexity in the planning problem by providing non-dominated solutions in a single and multi-objective model. The presented method in this paper is a static model and assumes that all investments are done at the beginning of the planning horizon. The decisions regarding the type, location, and capacity of new equipment are determined by the distribution system owner. The paper is organized as follows: Problem formulation is presented in Section 2. Optimization method and solution algorithm are explained in section 3. The implementation of the proposed algorithm on the DSP problem and results are shown in the section 4. This paper is concluded in the final section.
$V_i$ Voltage of the i-th node (p.u.)

2.1.3. Total System Losses (TSL)

Total system losses can be expressed as:

$$ J_{TSL} = \sum_{i=1}^{N} R_{i} |I_i|^2 $$

(7)

Where $J_{TSL}$ Objective function of the total system losses

2.2. Problem Constraints

2.2.1. Distribution power flow equations

This constraint is met by load flow calculation [21], considering the cost of available power sources and dispatching them. In this approach, DG units are assumed as negative loads.

$$ \sum_{pq=1}^{N} P_i^D = \sum_{pq=1}^{N} P_i^D + \sum_{pq=1}^{N} \sum_{j=1}^{M_{DG}} S_i^{DG} p_{ij}^{DG} \sigma_{ij}^p - \sum_{pq=1}^{N} X_i |I_i|^2 $$

(8)

$$ \sum_{pq=1}^{N} Q_i^D = \sum_{pq=1}^{N} Q_i^D + \sum_{pq=1}^{N} \sum_{j=1}^{M_{DG}} S_i^{DG} q_{ij}^{DG} \sigma_{ij}^q - \sum_{pq=1}^{N} X_i |I_i|^2 $$

(9)

Where

$Q_i^D$ Total reactive power transferred from electricity market in the i-th substation (MVAR)
$P_i^D$ Total active power consumption in the i-th node (MW)
$Q_i^D$ Total reactive power consumption in the i-th node (MVAR)
$X_i$ Reactance of the i-th feeder

2.2.2. Distribution feeder limit

This limit takes into consideration the new investments in feeder upgrade:

$$ S_i \leq S_{i}^{MAX} \quad l = 1,2,3,\ldots,L $$

(10)

Where

$S_i$ Power transferred from the l-th feeder (MVA)
$S_{i}^{MAX}$ Thermal capacity limit of the l-th feeder (MVA)

2.2.3. Substation and DG capacity limits

The power delivered by substation and DG unit must be less than their capacity [22].

$$ S_i^{Tr} \leq S_{i}^{Tr,MAX} \quad \forall i \in N $$

(11)

$$ S_i^{DG} \leq S_{i}^{DG,MAX} \quad \forall i \in M $$

(12)

Where

$S_i^{DG}$ Total power generated from DG in the i-th load node(MVA)
$S_i^{DG,MAX}$ DG capacity limit in the i-th load node (MVA)
$S_i^{Tr}$ Total power transferred from electricity market in the i-th substation (MVA)
$S_i^{Tr,MAX}$ Maximum substation capacity limit in the i-th substation (MVA)

2.2.4. Voltage limits

Voltage value of each node should remain within acceptable limits:

$$ V_i^{MIN} \leq V_i \leq V_i^{MAX} $$

(13)

Where

$T N$ Total number of system nodes
$V_i^{MIN}$ Minimum acceptable voltage of the i-th node (p.u.)
$V_i^{MAX}$ Maximum acceptable voltage of the i-th node (p.u.)

2.2.5. Maximum DG penetration

The total DG capacity is considered less than 30% of the total load.

3. Optimization Method and Solution Algorithm

3.1. Binary Particle Swarm Optimization (PSO)

PSO is an evolutionary algorithm that shows the social behaviour of birds or a fish school [23]. This algorithm was originally developed for nonlinear optimization problems with continuous variables. However, it could be easily expanded to handle problems with discrete or binary variables [24]. In the original PSO algorithm each particle is updated by a velocity vector. The velocity vector can be affected by three parameters: its own previous best value called pbest and the best position of all particles called gbest. The inertia weight factor is used to control the impact of the previous value of velocities on the current velocity. The velocity and position vector are updated as follows [25]:

$$ V_i^{iter+1} = w \cdot V_i^{iter} + C_1 \cdot r_1 \cdot (X_{pbest.i} - X_i^{iter}) + C_2 \cdot r_2 \cdot (X_{gbest,pq} - X_i^{iter}) $$

(14)

$$ X_{pbest.i} = X_{pbest,q} + V_{pq}^{iter} $$

(15)

Where

$\text{iter}$ Iteration counter
$p$ Particle counter
$q$ Decision variable counter
$v$ Particle velocity vector
$x$ Particle position vector
$w$ Inertia weight factor $\in [0.4, 0.9]
$c_1, c_2$ Acceleration (learning) factors
$r_1, r_2$ Random numbers $\in [0, 1]
$p_{pbest}$ Personal best position
$p_{gbest}$ Global best position

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The binary PSO algorithm (BPSO) has similar structure. The velocity is probability of a bit to be 0 or 1 and the position vector of the particle is a vector of binary digits, rather than a vector of continuous values [26]:

\[
X_{pq} = \begin{cases} 
1 & \text{rand}(l) < S(V_{pq}) \\
0 & \text{otherwise}
\end{cases}
\]  

(16)

where \(S(x)\) is the sigmoid function:

\[
S(x) = \frac{1}{1 + \exp(-x)}
\]  

(17)

When velocity values are too large or small the probability of change is zero. For zero velocity, the sigmoid function yielded a probability equal to 0.5 [27, 28].

3.2. The Proposed Algorithm for Single-Objective Optimization

All new transformers and DG units are modeled in a binary vector and illustrated in Fig. 1. This particle position vector in BPSO is a string of 0 and 1 bits and shows decision variables \((\sigma_{pq}^N, \sigma_{pq}^M, \sigma_{pq}^{T}.\) In this vector \(n(N)\) shows substations and each string has \(N_{tr}\) bits. The other part shows the set of load nodes which DG units can be installed on it. This part has \(n(M)\) strings and each string has \(M_{dg}\) bits. So the binary decision variable vector has \(m\) bits which equals to the total possible transformers and DGs.

\[
m = n(N) \cdot N_{tr} + n(M) \cdot M_{dg}
\]  

(18)

Fig. 1. Binary vector of particle position.

When the binary vector of particle position is generated, this vector is decoded and summation of DG units or transformers in each string in the corresponding node is obtained. Fig. 2 shows the flowchart of the proposed method.

3.3. Multi-Objective Optimization with Weighting Sum Method

Multi-objective optimization method simulates systems more accurate than single-objective optimization. This method is usually characterized by conflicting goals and gives the planner the capability of making the final decision based on individual point of view. The planner or decision-maker can trade-off among suitable solutions.

Since DGs have several benefits simultaneously, DSP problem in the presence of DGs can be considered as a multi-objective optimization problem. This paper uses a weights sum method (WSM) [29] to solve this problem. In this method the objective functions are combined to a single objective [28, 30]:

\[
\min \ f(x) = \min \ \sum_{i=1}^{k} w_i f_i(x)
\]  

(19)

The set of optimal solutions named Pareto optimal solution is obtained by changing the weighting factors [31, 32]. Since the scale of objective functions is different, it is required to normalize them [30, 32] as follows:

\[
\overline{J}_i(x) = \frac{J_i(x) - J_{i,min}}{J_{i,max} - J_{i,min}}
\]  

(21)

In this paper, three objectives are considered as multi-objective problem (Fig. 3). WSM yields the multi-objective function as follows:

\[
J_{MO} = w_1 \cdot \overline{J}_{REC} + w_2 \cdot \overline{J}_{TVD} + w_3 \cdot \overline{J}_{TSL}
\]  

(22)
4. Case Study and Numerical Results

To validate the proposed method, it has been applied to two different cases. To validate the result, the problem has been solved by two evolutionary algorithms, binary GA and binary PSO. These algorithms have been run several times and the best solution has been presented. The first case is 9-bus test system and the other is 30-bus system.

4.1. Assumption

In this survey, without loss of generality, only gas turbines are considered as DG units. It is supposed that the investment cost of gas turbine is 0.5 MS/MVA and the operating cost is 70 S/MWh. The capacity of new transformers for the main substation is 10 MVA, the investment cost of each transformer is 0.2 MS and the operating cost in the electricity market for purchasing power from the main grid is considered to be 70 S/MWh. The cost of reinforcement for each feeder is 0.15 MS/km. The planning horizon is assumed 4 years and the discount rate is considered to be 12.5% [20].

4.2. Single objective simulations

In this section, the DSP problem is treated as a single objective optimization problem and objective functions are optimized separately.

4.2.1. Case 1: 9-bus distribution system

Fig. 4 shows the first case study which is a 132/33 kV 9-bus system. The capacity of substation transformer and the thermal capacity of feeders are 40 and 12 MVA respectively. Forecasted load growth is 28 % at the planning horizon. The capacity of DG units in this case study, are assumed 1 MVA. The maximum number of the DG units in each bus is limited to 4 units plus one DG as a backup unit for scheduled maintenance intervals. Voltage value at each node should be remained in [0.95, 1.05]. The other data of this system can be found in Appendix [33].

4.2.2. Case 2: 30-bus distribution system

This paper investigates two scenarios for each case study to show the effectiveness of DG effect on the DSP problem.

- Scenario 1: In this scenario the DSP problem considers substation reinforcement and feeders’ replacement without DG units.
- Scenario 2: The proposed DSP problem considers the DG option integrated with the substation reinforcement and feeders’ replacement.

Table 1 and Table 2 show numerical results for objectives and decision variables in case 1.

| Table 1. Numerical results for objectives (case 1) |
|-------------------------------|-------------------------------|-------------------------------|
| Without | Objective function | DG | TEC | TVD | TSL |
| Total Expansion Cost (MS) | 92.4411 | 93.8073 | 97.0657 | 97.0657 |
| Total Voltage Deviation (per unit) | 0.36986 | 0.30598 | 0.2223 | 0.2223 |
| Total system Losses (MW) | 1.5077 | 1.0011 | 0.5987 | 0.5987 |

| Table 2. Numerical results for decision variables (case 1) |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Without DG | TEC | TVD | TSL |
| $i$ | $\sigma_{Di}^{Tr}$ | $\sigma_{Di}^{DG}$ | $l$ | $\sigma_{Si}$ | $\sigma_{Si}^{Tr}$ | $\sigma_{Si}^{DG}$ | $l$ | $\sigma_{Qi}$ | $\sigma_{Qi}^{Tr}$ | $\sigma_{Qi}^{DG}$ | $l$ |
| 1 | 2 | - | - | 1 | 1 | 1 | 1 | - | - | 3 | 4 | 1 | 1 | - | - | 3 | 4 | 1 | 1 |
| 2 | - | - | - | 3 | 1 | - | - | 3 | 4 | - | - | 5 | 4 | - | - | - | - | 5 | 4 | - | - |
| 3 | - | - | - | 7 | 1 | - | - | 7 | 4 | - | - | - | - | 7 | 4 | - | - | - | - |
| 4 | - | - | 9 | 2 | - | - | 9 | 4 | - | - | - | - | 9 | 4 | - | - | - | - | - | - |

4.2.2. Case 2: 30-bus distribution system

Table 1 and Table 2 show numerical results for objectives and decision variables in case 1.
The second case study is shown in Fig. 5. It is an 11 kV distribution system with 30 nodes and substation node. The system has one main feeder and three laterals. The capacity of the substation transformers and the thermal capacity of feeders are 12 MVA. Total demand of the system is 10.224 MVA and forecasted power demand will be approximately 13.291 MVA after 4 years. The size of the DG units is multiple of 0.1 MVA and the upper limitation of the DG capacity at each node is 0.3 MVA. The new transformer units used in substation expansion is a 10 MVA transformer. Voltage value at each node should be remained in [0.9, 1.1]. The other characteristics data of this system can be found in [34, 35].

![Fig. 5. 30-bus distribution system](image)

Also in the second case study, numerical results for objectives and decision variables for each optimal solution are shown in Table 3 and Table 4. In this case also two mentioned scenarios are considered. As can be seen in the Tables 3, the result of TVD objective function is identical to TSL. So in DG planning two different solutions are obtained: first one belongs to TEC and the other one belongs to TVD and TSL. It is clear that the voltage profile in DG option solutions is better than the voltage profile in the case without DG, consequently total system losses is decreased and feeders upgrading is not necessary. These improvements are the benefits of using DG in the DSP problem.

### Table 3. Numerical results for objectives (case 2)

<table>
<thead>
<tr>
<th>Total Expansion Cost (M$)</th>
<th>Without DG</th>
<th>Objective function</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total TVD</td>
<td>TVD</td>
</tr>
<tr>
<td></td>
<td>TSL</td>
<td></td>
</tr>
<tr>
<td></td>
<td>23.7136</td>
<td>24.0774</td>
</tr>
<tr>
<td></td>
<td>24.7431</td>
<td></td>
</tr>
<tr>
<td>Total Voltage Deviation (per unit)</td>
<td>2.8609</td>
<td>1.9279</td>
</tr>
<tr>
<td>Total system Losses (MW)</td>
<td>1.2689</td>
<td>0.8043</td>
</tr>
<tr>
<td></td>
<td>0.5144</td>
<td>0.5144</td>
</tr>
</tbody>
</table>

Also the binary GA is applied to the DSP problem. Fig. 6 depicts the convergence plot of BGA and BPSO algorithms for case 2 in the second scenario. It is clear that the proposed algorithm is faster than BGA.

![Fig. 6. Convergence plot of BGA and BPSO for case 2 in scenario 2](image)

### 4.3. Multi-Objective Results

In this section, the DSP problem in the presence of DGs is treated as a multi-objective optimization problem, and all three objective functions are optimized simultaneously.

#### 4.3.1. Case 1: 9-bus distribution system

At first, 231 combinations of weighing factors are generated with 0.05 increments in the case 1. Then for each combination the problem is solved. After eliminating identical solutions, 6 non-dominated solutions are obtained. Each solution is shown in Table 5 with its corresponding items including total expansion cost, total voltage deviation, total system losses, location, and the size of DGs.

#### 4.3.2. Case 2: 30-bus distribution system

In this case, 66 combinations of weighing factors are generated with 0.01 increments. Then for each one, problem is solved. After eliminating identical solutions, 5 non-dominated solutions are obtained in this case. Each solution and its corresponding items, especially objective functions and decision variables, are shown in Table 6.

It is important that each Pareto-optimal solution is a choice for a system planner. Indeed, after obtaining the Pareto-optimal solutions, the decision maker needs to choose the best solution based on the importance of the different objectives and the budget constraints.

It can be seen that when each objective reaches its optimum value, other objectives will keep out their optimum in comparison with single objective optimization. The importance of the objective functions is considered by weight factors. For example, in case 1, when w1, w2 and w3 are 0.4, 0.35 and 0.25 respectively, total expansion cost, total voltage deviation and total system losses are 96.45 M$, 0.2323 p.u. and 0.6444 MW.
Table 5. Multi-Objective numerical results in (case 1)

<table>
<thead>
<tr>
<th>Solution 1</th>
<th>Solution 2</th>
<th>Solution 3</th>
<th>Solution 4</th>
<th>Solution 5</th>
<th>Solution 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>w1</td>
<td>1</td>
<td>0.55</td>
<td>0.5</td>
<td>0.4</td>
<td>0.35</td>
</tr>
<tr>
<td>w2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.35</td>
<td>0</td>
</tr>
<tr>
<td>w3</td>
<td>0</td>
<td>0.45</td>
<td>0.5</td>
<td>0.25</td>
<td>0.65</td>
</tr>
<tr>
<td>J_{TEC} (MS)</td>
<td>93.807</td>
<td>94.784</td>
<td>95.184</td>
<td>96.45</td>
<td>96.434</td>
</tr>
<tr>
<td>J_{TVD} (p.u.)</td>
<td>0.3059</td>
<td>0.2742</td>
<td>0.2652</td>
<td>0.2323</td>
<td>0.2342</td>
</tr>
<tr>
<td>J_{TSL} (MW)</td>
<td>1.0011</td>
<td>0.8256</td>
<td>0.7176</td>
<td>0.6444</td>
<td>0.6357</td>
</tr>
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</table>

Table 6. Multi-Objective numerical results in (case 2)

<table>
<thead>
<tr>
<th>Solution 1</th>
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</thead>
<tbody>
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<td>w1</td>
<td>1</td>
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<td>w2</td>
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<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>w3</td>
<td>0</td>
<td>0.3</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>J_{TEC} (MS)</td>
<td>24.077</td>
<td>24.094</td>
<td>24.399</td>
<td>24.64</td>
</tr>
<tr>
<td>J_{TVD} (per unit)</td>
<td>1.9279</td>
<td>1.9058</td>
<td>1.7068</td>
<td>1.5854</td>
</tr>
<tr>
<td>J_{TSL} (MW)</td>
<td>0.8043</td>
<td>0.78623</td>
<td>0.6259</td>
<td>0.53996</td>
</tr>
<tr>
<td>Trans.</td>
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<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>DG</td>
<td>26</td>
<td>27</td>
<td>36</td>
<td>42</td>
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<tr>
<td>B-DG</td>
<td>9</td>
<td>9</td>
<td>12</td>
<td>14</td>
</tr>
<tr>
<td>Node</td>
<td>Number of DGs</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2-7</td>
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<td>0</td>
<td>0</td>
<td>0</td>
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It is obvious that by increasing the weight factor, the significance of relevant objective function is raised and consequently its value is optimized more than previous.

5. Conclusions

This paper provides a binary particle swarm optimization algorithm for non-linearity and non-convexity DSP problem including DG in the single and multi-objective case. The goal of the proposed DSP problem was total expansion cost, total voltage deviation, and total system losses. The algorithm has been successfully applied to the problem and according to the obtained results the points can be highlighted as follows:

- In the proposed formulation, conventional distribution system expansion planning problem is integrated with DG placement problem without any approximation.
- Since the original version of evolutionary algorithms cannot solve integer problem the presented optimization algorithm based on binary GA and PSO can obtain better results.
- The other salient advantage of the proposed scheme is offering several non-dominated solutions which allow the decision maker to use the best solution based on the importance of the different objectives and the budget constraints.

6. Appendix: 9-Bus Test System Data

Table 7 shows the data of system [33].

Table 7. 9-bus system feeder’s data

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<th>To</th>
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<th>Reactance (Ω/Km)</th>
<th>Length (Km)</th>
<th>MVA</th>
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References


